Bose-Einstein correlations in the Quantum Clan Model

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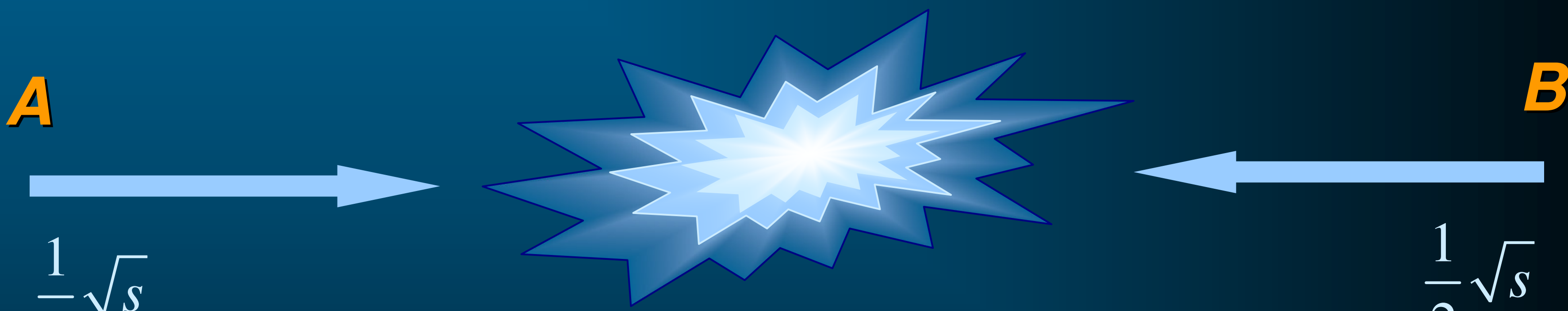
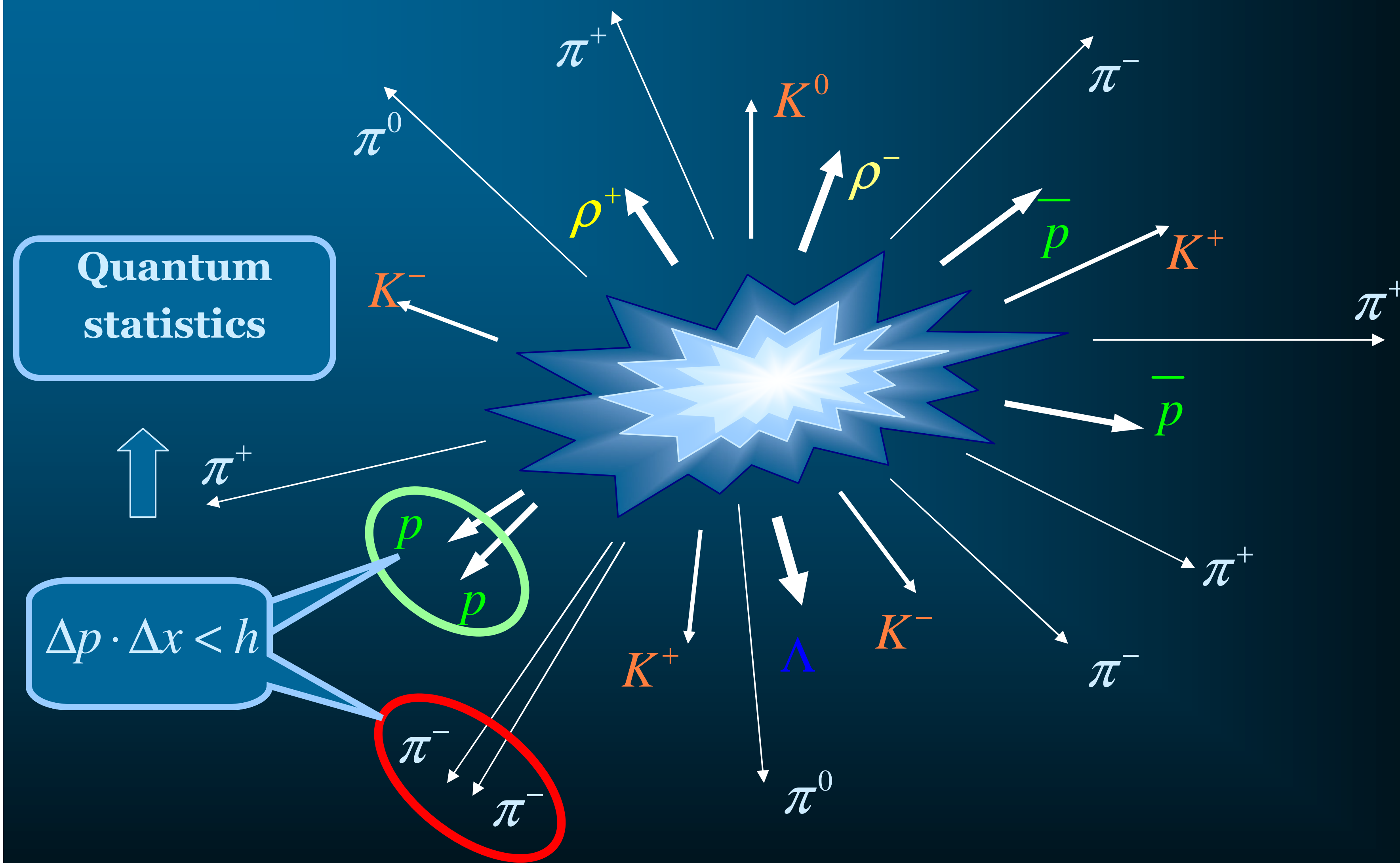
High-Energy collisions …

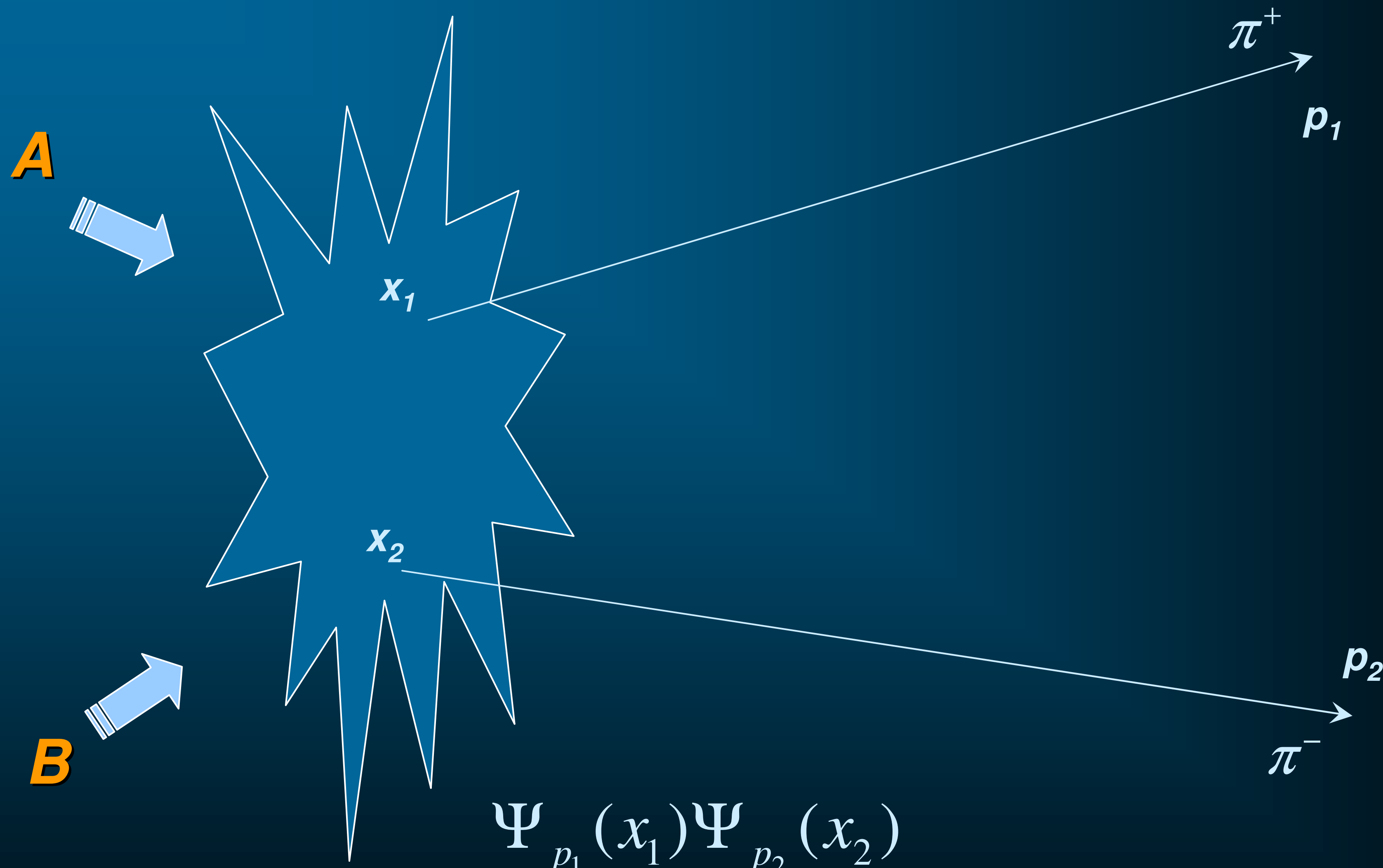


$$\frac{1}{2}\sqrt{s} \qquad \frac{1}{2}\sqrt{s}$$

High-Energy collisions ...

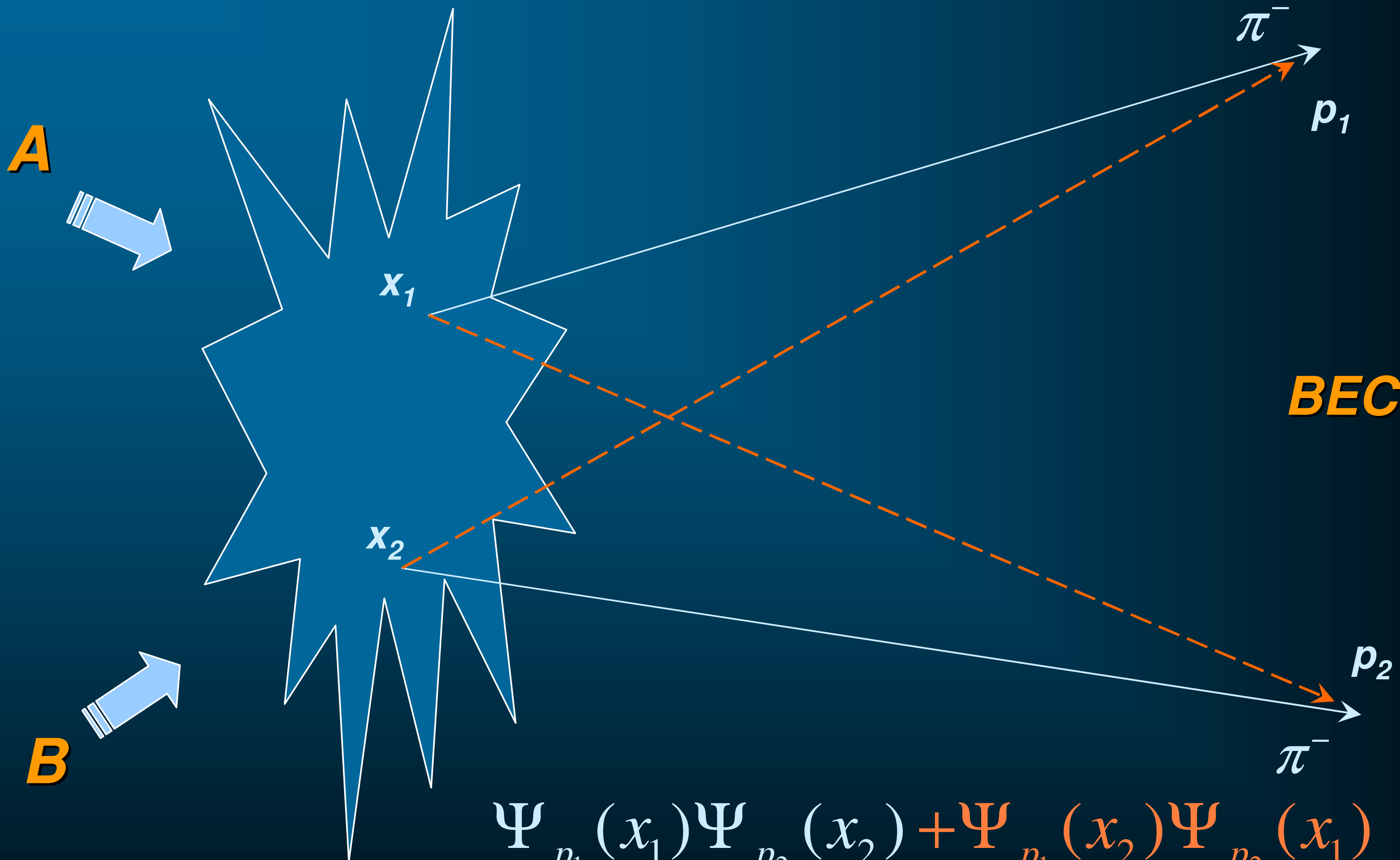


Defenition



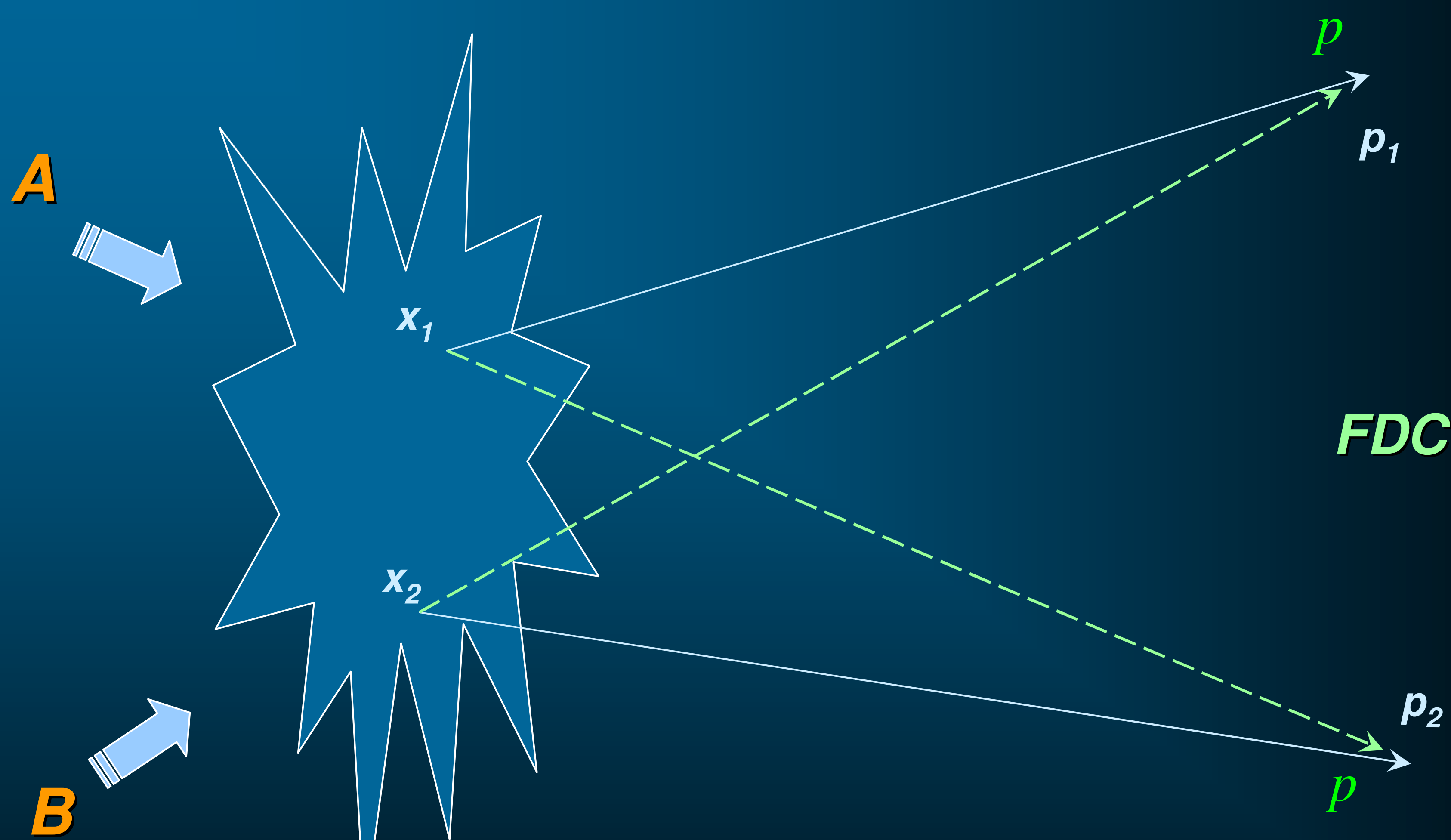
$$\Psi_{p_1}(x_1)\Psi_{p_2}(x_2)$$

Defenition BE



$$\Psi_{p_1}(x_1)\Psi_{p_2}(x_2) + \Psi_{p_1}(x_2)\Psi_{p_2}(x_1)$$

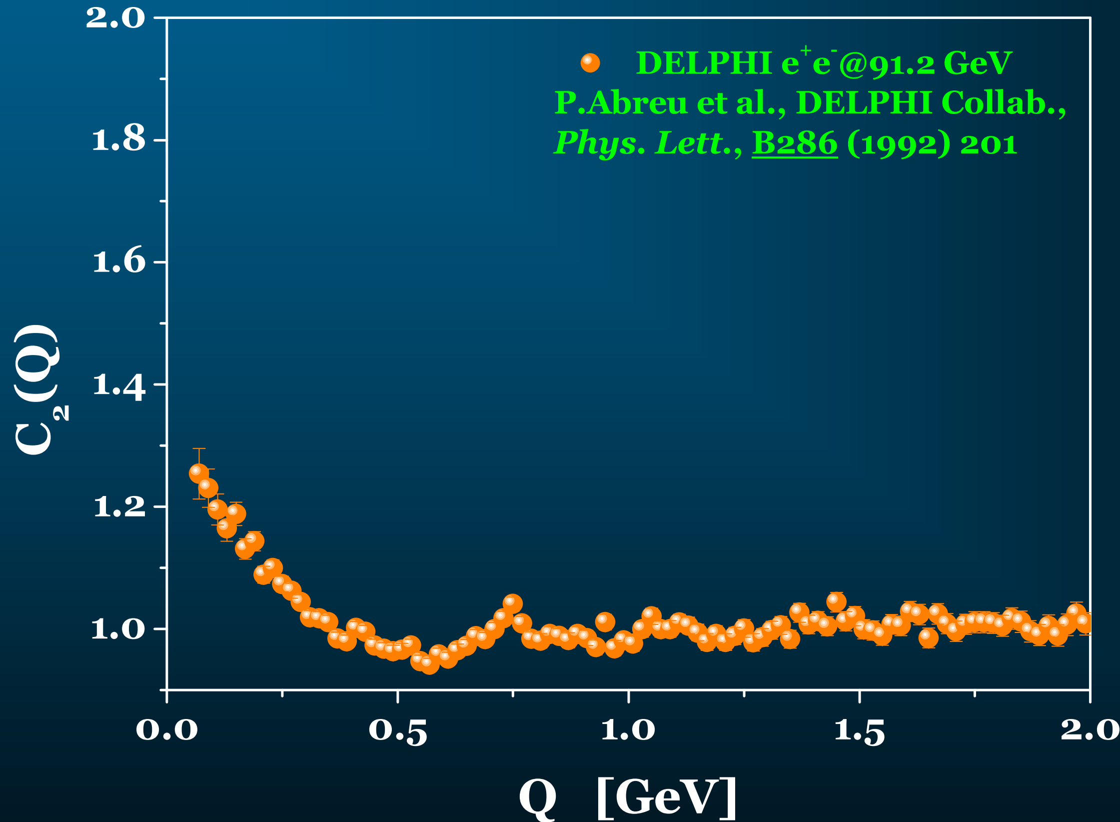
Defenition FD



$$\Psi_{p_1}(x_1)\Psi_{p_2}(x_2) - \Psi_{p_1}(x_2)\Psi_{p_2}(x_1)$$

BEC Data

$$C_2(Q=|p_1-p_2|) \equiv \frac{N_2^{BE}(p_1,p_2)}{N_2^{ref}(p_1,p_2)}$$



Why measure BEC ?..

- To determine the space-time development of a boson production region.

- The influence of BEC on the measurement of the W mass at LEP2

- Higher-order correlation effects and their consequences.

Why BEC ?

Source size measurement

$$C_2(Q = |p_1 - p_2|) \equiv \frac{N_2^{BE}(p_1, p_2)}{N_2^{ref}(p_1, p_2)} \xrightarrow{usually} \frac{N_2^{BE}(p_1, p_2)}{N_1(p_1)N_1(p_2)}$$

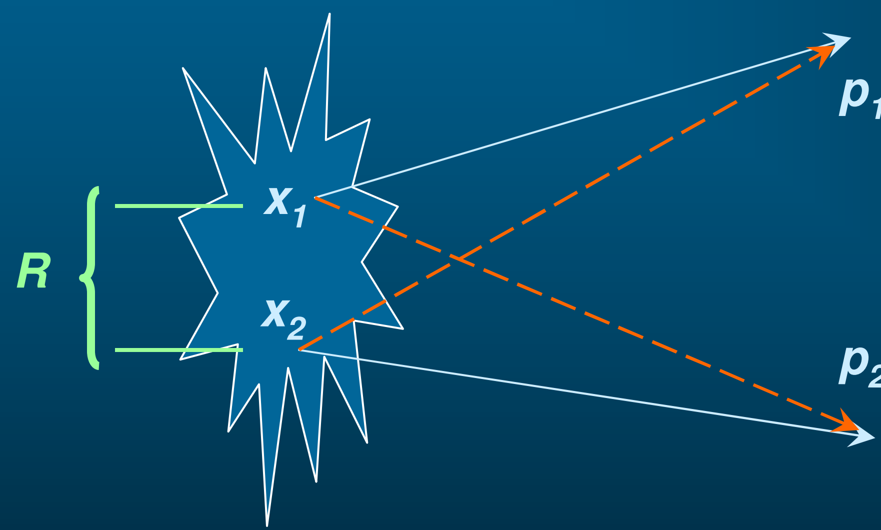
$$\rho(x_1, x_2) = \rho(x_1)\rho(x_2)$$

$$C_2(Q) = 1 + \left| \int d^4 x \rho(x) e^{iQx} \right|^2 \Longrightarrow 1 + \quad |\tilde{\rho}(QR)|^2$$

Why BEC ?

Source size measurement

$$C_2(Q = |p_1 - p_2|) \equiv \frac{N_2^{BE}(p_1, p_2)}{N_2^{ref}(p_1, p_2)} \xrightarrow{usually} \frac{N_2^{BE}(p_1, p_2)}{N_1(p_1)N_1(p_2)}$$



R – source size

Fourier transformation of hodronic source

$$C_2(Q) = 1 + \left| \int d^4x \rho(x) e^{iQx} \right|^2 \Longrightarrow 1 + \lambda \left| \tilde{\rho}(QR) \right|^2$$

coherence

$$0 < \lambda < 1 \Rightarrow 1 < C_2(Q) < 2$$

What one should remember…

- *all final state interactions (like Coulomb and Strong) are neglected*
- *all possible correlations inside source are neglected*

$$\rho(x_1,\ldots,x_N) \xrightarrow{factorization} \prod_{i=1}^{N} \rho(x_i)$$

- *momenta of particles are detected (not position of production points)*

Why BEC ?

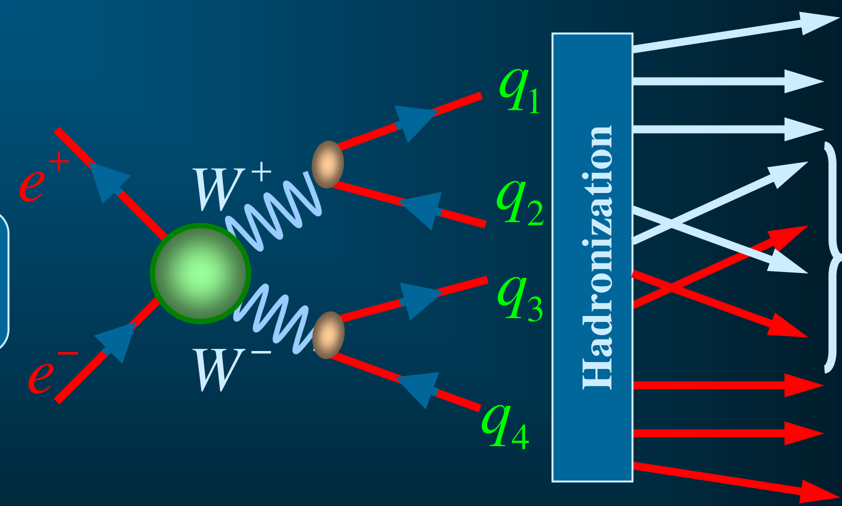
W mass measurement

Hadronization region ~ 0.5 fm

Separation of W-pair ~ 0.1 fm

Overlap between the two production regions

BEC between bosons from different W's (inter-W BEC)?



inter-W BEC

Potential bias of measured M_W

$\Delta M_W \sim 40\text{-}50$ MeV

Why BEC ?

W mass measurement

Hadronization region ~ 0.5 fm

Separation of W-pair ~ 0.1 fm

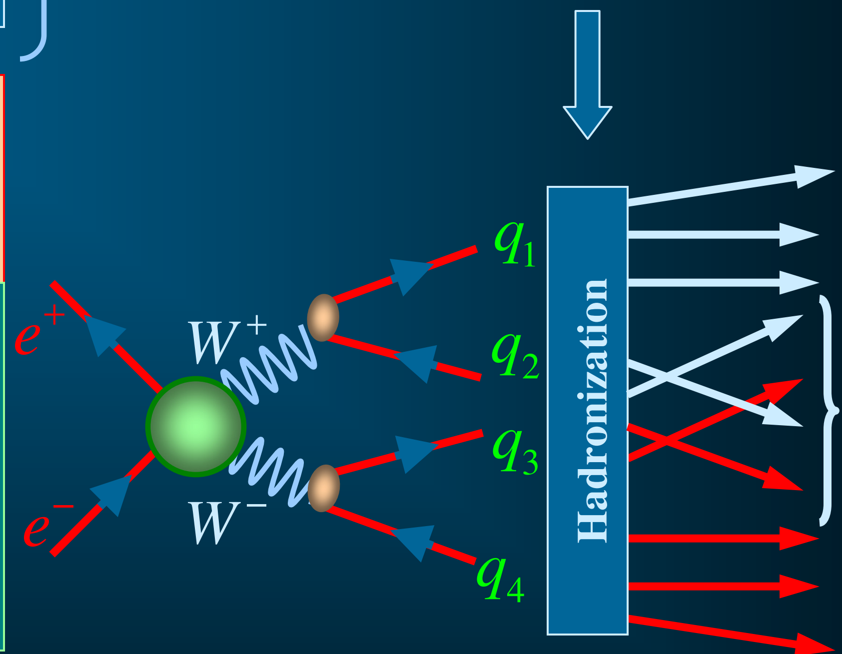
Overlap between the two production regions

NO

L3, OPAL, ALEPH –
no indication for inter-W BEC

YES

DELPHI-
indication for inter-W BEC
at the 2.4σ level
(Ignacio Aracena, talk at ICHEP'04)



inter-W BEC

$\Delta M_W \sim$ 40-50 MeV

Potential bias of measured M_W

Why BEC ?

Multiparticle correlation measurement

- Three-particle correlation are sensitive to asymmetries in particle source shape*.

- **asymmetric source**

$$\cos\phi = \frac{R_3^{genuine}(Q_{12},Q_{13},Q_{23})-1}{2\sqrt{(R_2(Q_{12})-1)(R_2(Q_{13})-1)(R_2(Q_{23})-1)}} \neq 1$$

- Combination of two- and three-particle correlation analyses give us a better handle on a degree of coherence, λ.

* V.L. Lyuboshitz, Sov. J. Nucl. Phys. 53 (1991) 514.

How to model it numerically?

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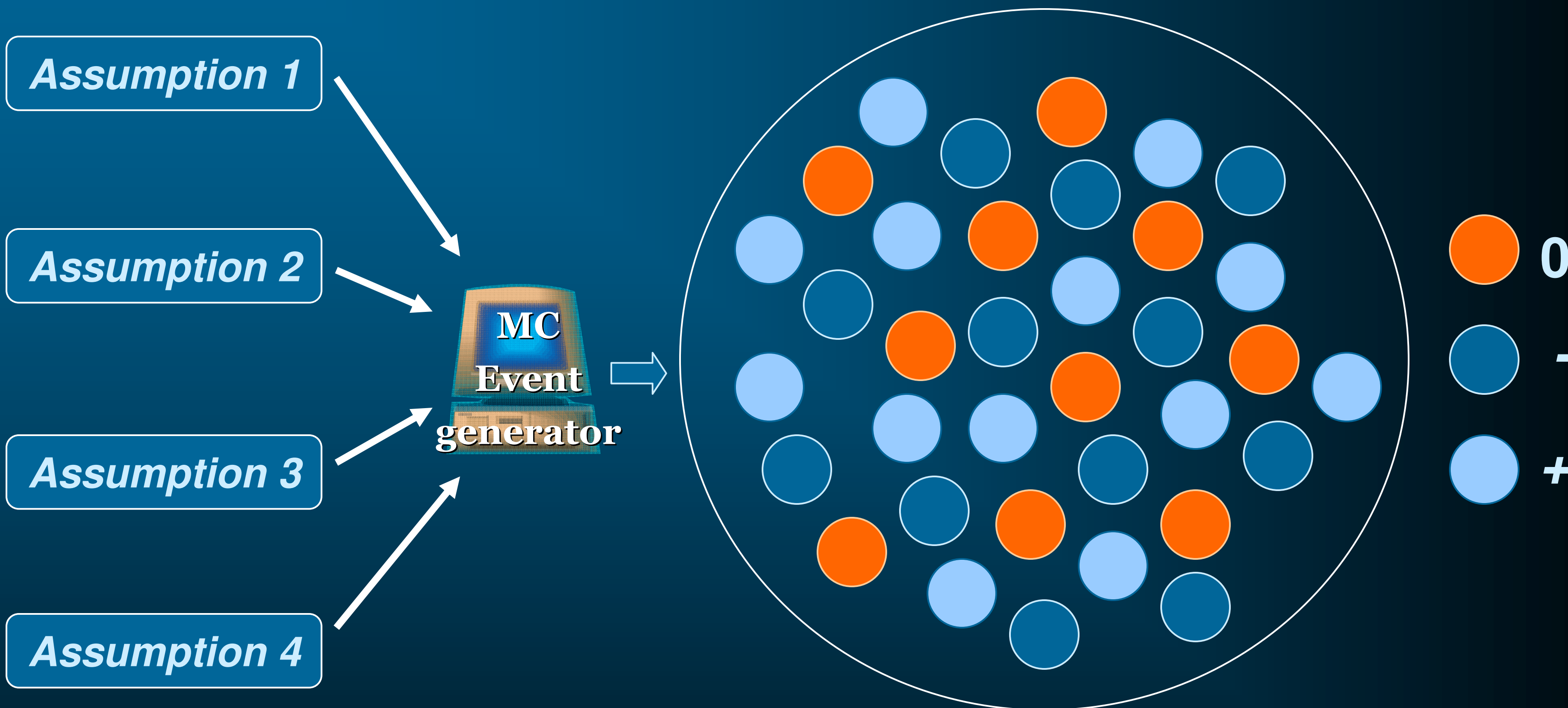


How to model it numerically?

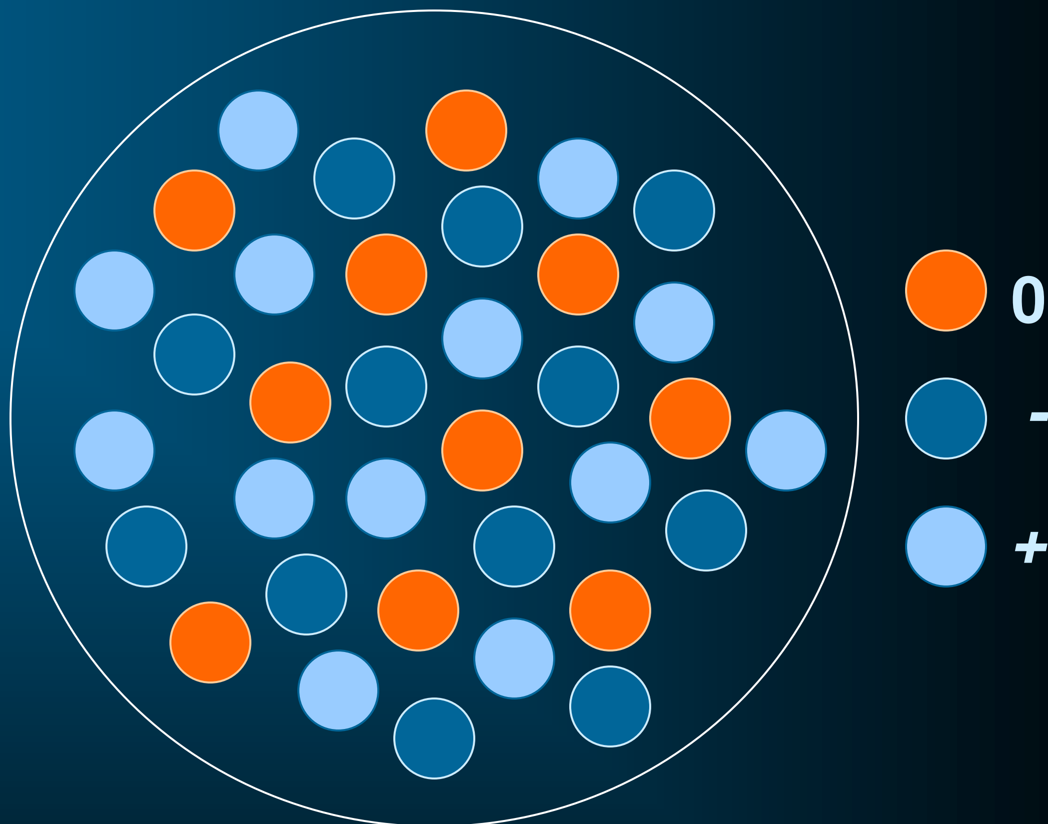
(a) **Momenta shifting***

$$dN(Q) \propto \frac{d^3p}{E} \propto \frac{Q^2 dQ}{\sqrt{Q^2+4m^2}}$$

$$\Downarrow$$

$$\int_0^Q \frac{q^2 dq}{\sqrt{q^2+4m^2}} =$$

$$\int_0^{Q+\delta Q} f_{BE}(Q) \frac{q^2 dq}{\sqrt{q^2+4m^2}}$$



$$f_{BE}(Q) \geq 1 \Longleftrightarrow \delta Q < 0$$

* L.Lönblad, T.Sjöstrand, *Eur.Phys.J.* **C2** (1998) 165

How to model it numerically?

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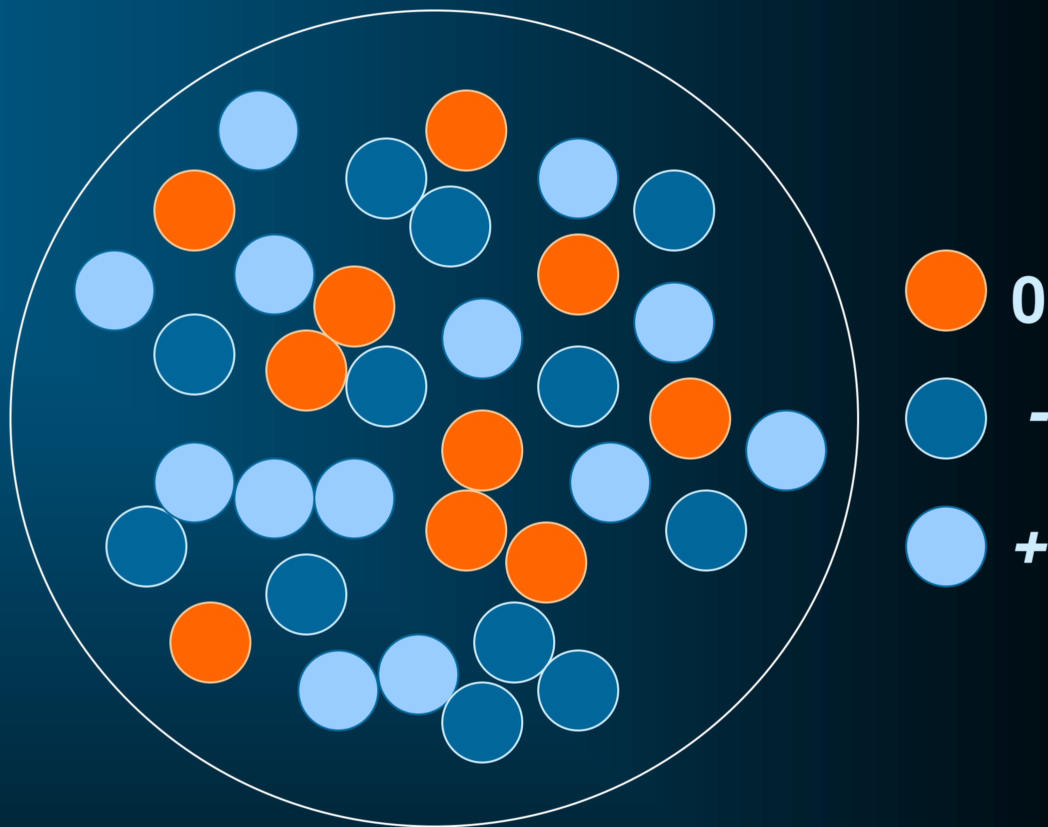

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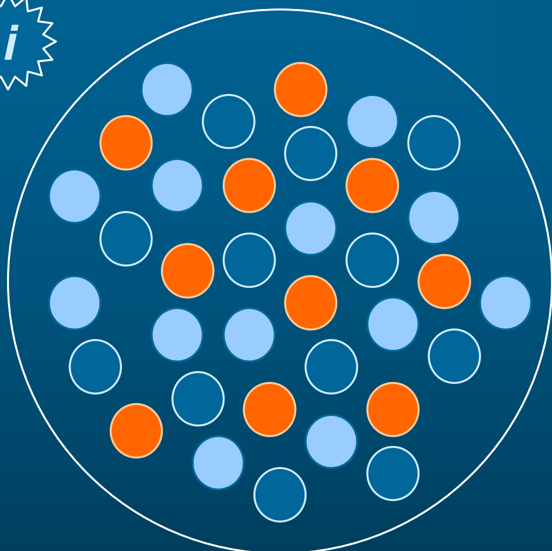
[*] L.Lönblad, T.Sjöstrand, *Eur.Phys.J.* **C2** (1998) 165

How to model it numerically?

(b) **weighting of events** *

i



$$W_e = \sum_{\{P(i)\}} \prod_{i=1}^{N_{event}} e^{-\frac{1}{2} Q_i^2 R_{input}^2}$$

$$\Downarrow$$

$$W_i < W_j$$

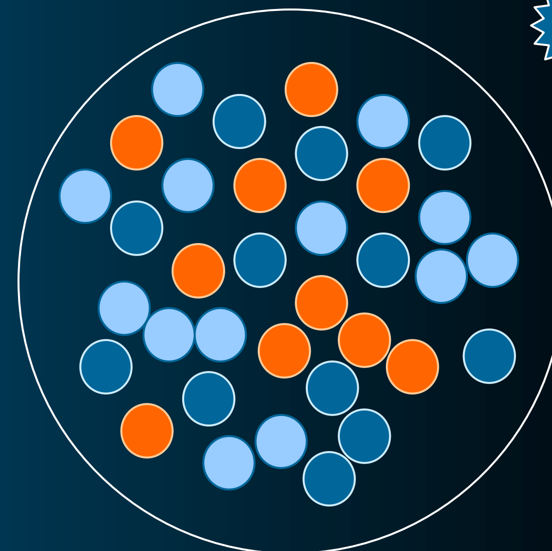
$$\Downarrow$$

for each E_i event one should take

$$\frac{W_j}{W_i} > 1 \quad \textbf{\textit{E}}_{\textbf{\textit{j}}} \textbf{\textit{ events}}$$

$$\{\ldots,\underbrace{E_i}_{n_i},\ldots,\underbrace{E_j}_{n_j},\ldots\}$$

j



$$\{\ldots,\underbrace{E_i}_{W_i n_i},\ldots,\underbrace{E_j}_{W_j n_j},\ldots\}$$

* K.Fiałkowski,R.Wit,J.Wosiek, *Phys.Rev.* **D57** (1998) 0940013

Problems ...

	(a) momenta shifting	*(b)* weighting of events
Energy-momentum conservation	*no*	*yes*
Single-particle distributions	*yes*	*no*

(a) and (b) ➡ *arbitrary function f(Q•R) has appeared:*

$$R_{input} \overset{often}{\rightleftarrows} R_{fit}$$ ➡ *Interpretation of R_{input} (R_{fit}) ???*

measure of correlation fluctuations ...

main idea

$$\langle n_i n_j \rangle = \langle n_i \rangle \langle n_j \rangle + \langle (n_i - \langle n_i \rangle) \cdot (n_j - \langle n_j \rangle) \rangle$$

$$= \langle n_i \rangle \langle n_j \rangle + \rho \sigma(n_i) \sigma(n_j)$$

σ(n) – dispersions of multiplicity distribution P(n)
ρ - the correlation coefficient

$$\rho = \begin{cases} +1, & \textit{Bose-Einstein} \\ 0, & \textit{Boltzmann} \\ -1, & \textit{Fermi-Dirac} \end{cases}$$

measure of correlation fluctuations ...

$$C_2(Q) = \frac{\langle n_i n_j \rangle}{\langle n_i \rangle \langle n_j \rangle} = 1 + \rho \frac{\sigma(n_i)\sigma(n_j)}{\langle n_i \rangle \langle n_j \rangle}$$

To get $\rho > 0 \; (\rho < 0)$ ***it is enough to:***

- ***select particle (from the pool of particles provided by MC event generator used)***
- ***allocate to it (randomly for BE or in some specific way for FD) charge (+/-/0)***
- ***and then allocate the same charge (in some prescribed way) to adjacent particles (in phase space) – for BE, for DF it is more complicated .....***

Boltzmann vs. Bose-Einstein

SYMMETRIZATION

$$\Psi_N = \prod_i \psi_i(x_i) \longrightarrow \Psi_N = \frac{1}{N!} \sum_{P\{i,j\}} \prod_i \psi_i(x_j)$$

POISSONIAN $\longrightarrow$ ***GEOMETRICAL***

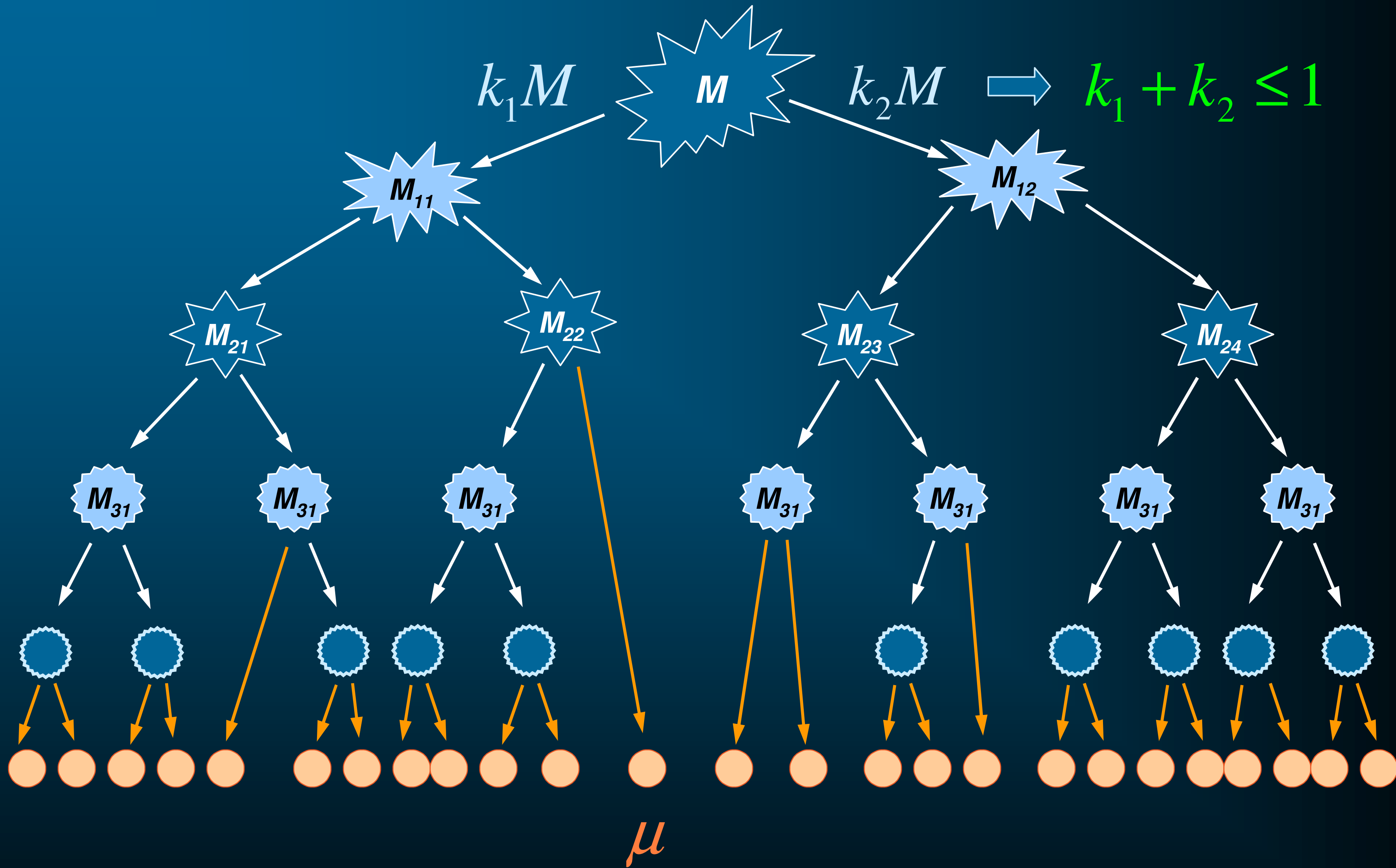
x N!

$$P_{Bltz}(N) = \frac{\nu^N}{N!} e^{-\nu} \qquad P_{BE}(N) = (1-\nu)\cdot\nu^N$$

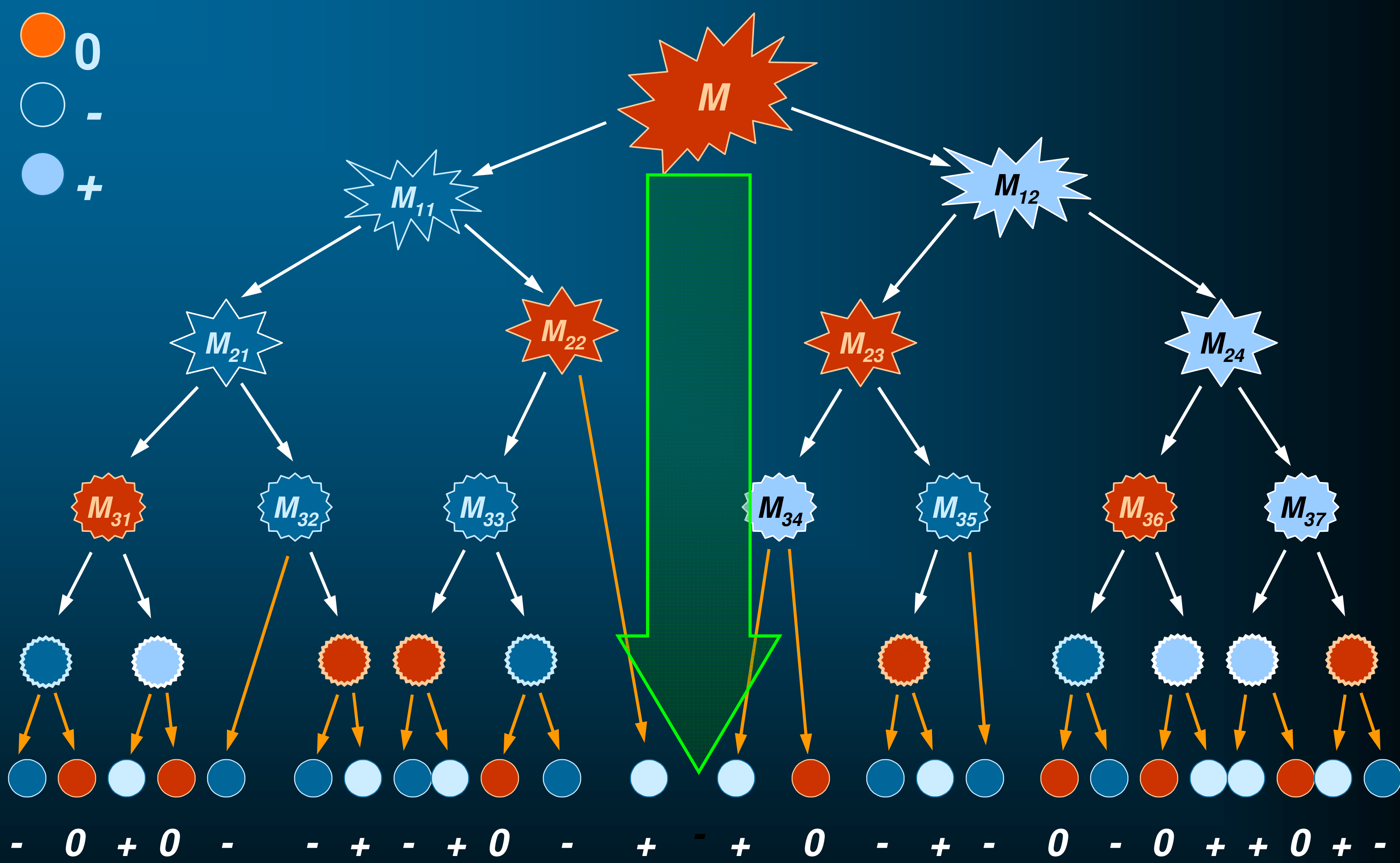
$$f_{Bltz}(p) = e^{-\frac{E}{kT}} \qquad f_{BE}(p) = \left[e^{\frac{E}{kT}} - 1 \right]^{-1}$$

- K.Zalewski, *Nucl. Phys. Proc. Suppl.* **74** (1999) 65
- A. Giovannini and H.B.Nielsen, *Proc. Of the IV Int. Symp. On Mult. Hadrodyn.*, Pavia 1973

Simple Cascade Model

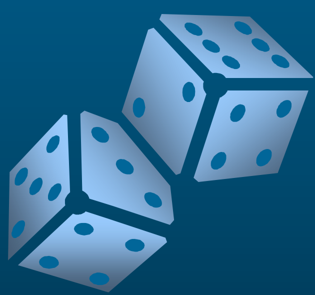


Cascade – charge flow ...



Cascade – algorithm application …

- 0 + 0 - - + - + 0 - + + 0 - + - 0 - 0 + + 0 + -

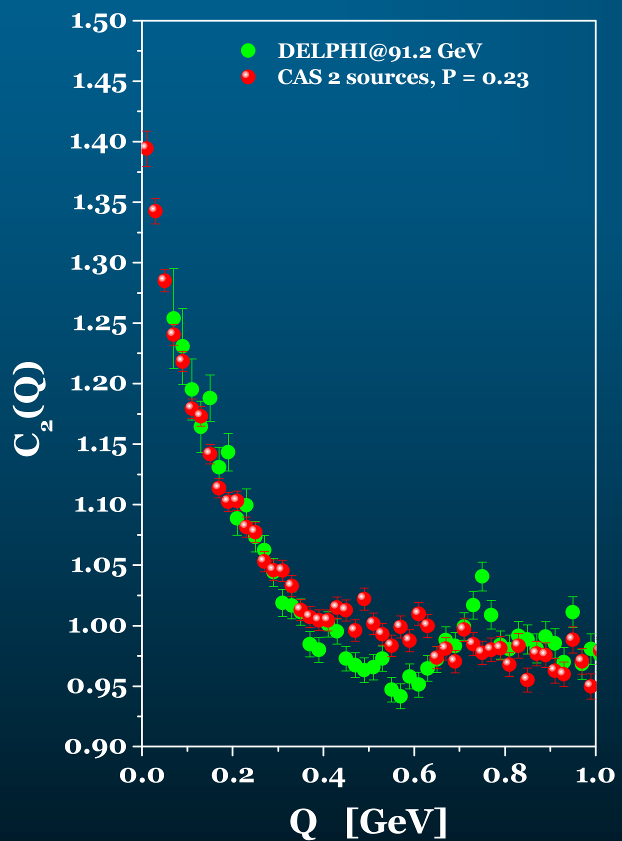


$$P = P_0 e^{-\frac{E}{kT}}$$

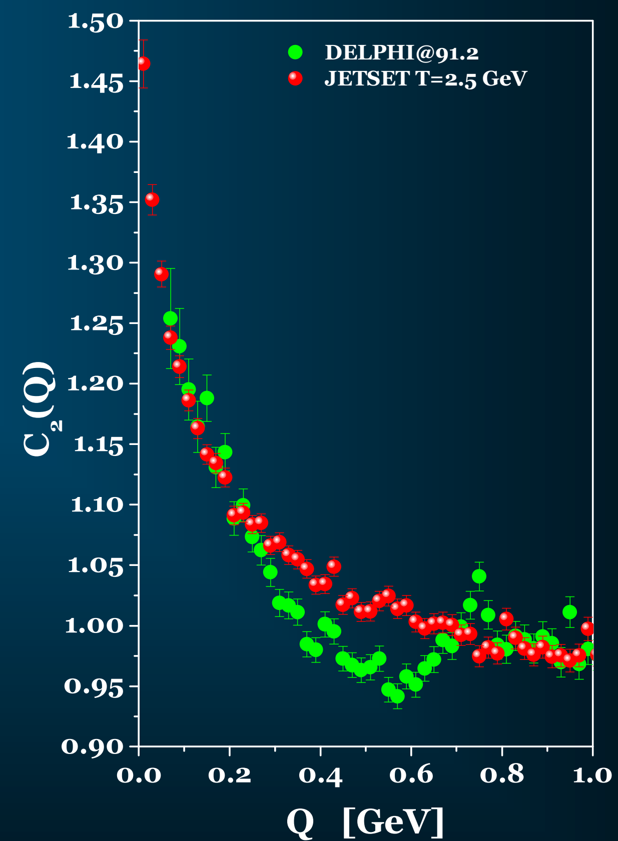
- - - - - 0 + + + + + - - 0 0 - + - + 0 0 + 0 + 0

$$\langle n_{cell} \rangle = \frac{P}{1-P} \quad \xrightarrow{\; P = e^{-\frac{E}{kT}} \;} \quad \langle n_{cell} \rangle = \frac{1}{e^{E/kT} - 1}$$

Results …



DELPHI@91.2 GeV
CAS 2 sources, P = 0.23
$C_2(Q)$
Q [GeV]

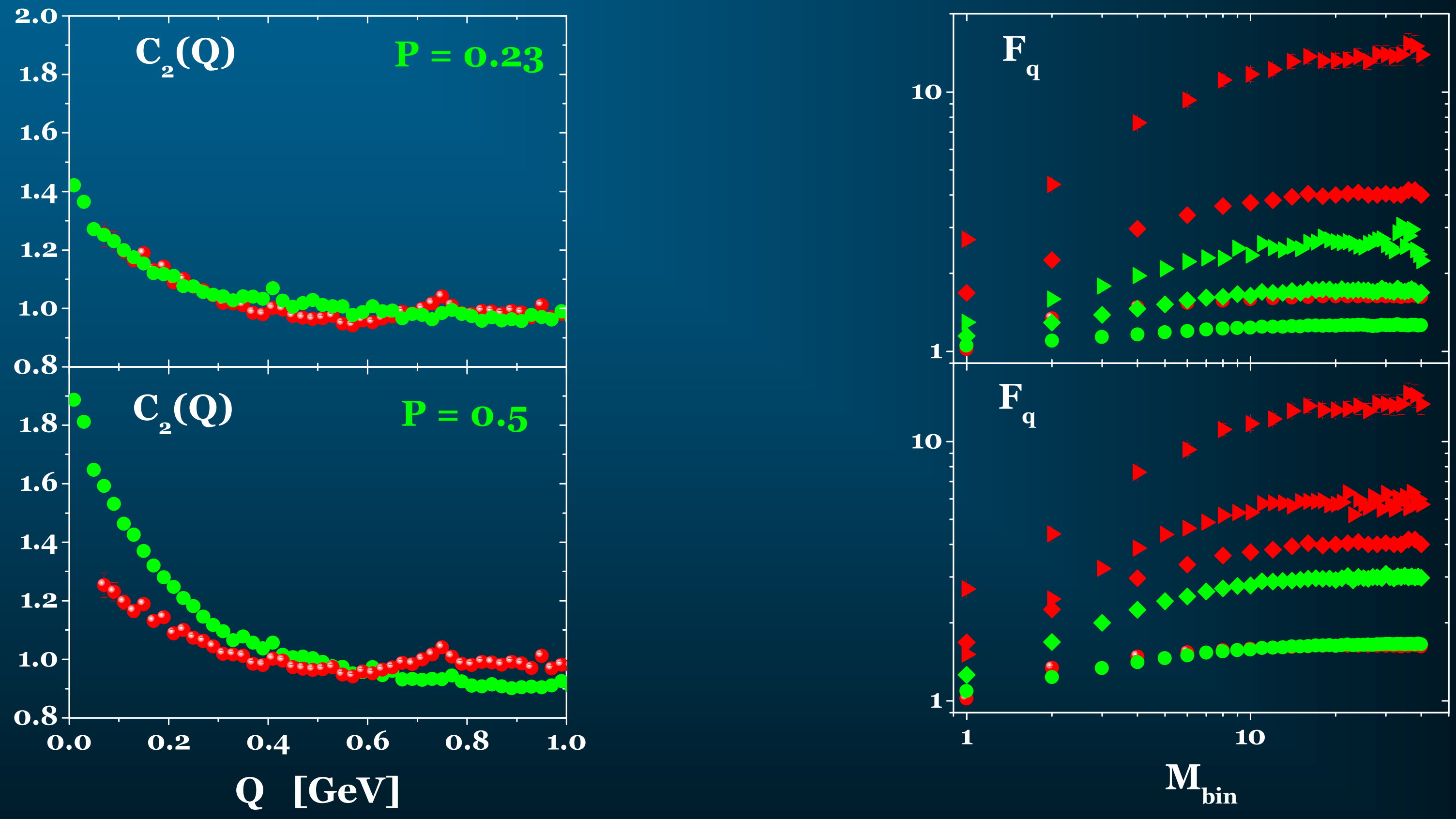


DELPHI@91.2
JETSET T=2.5 GeV
$C_2(Q)$
Q [GeV]

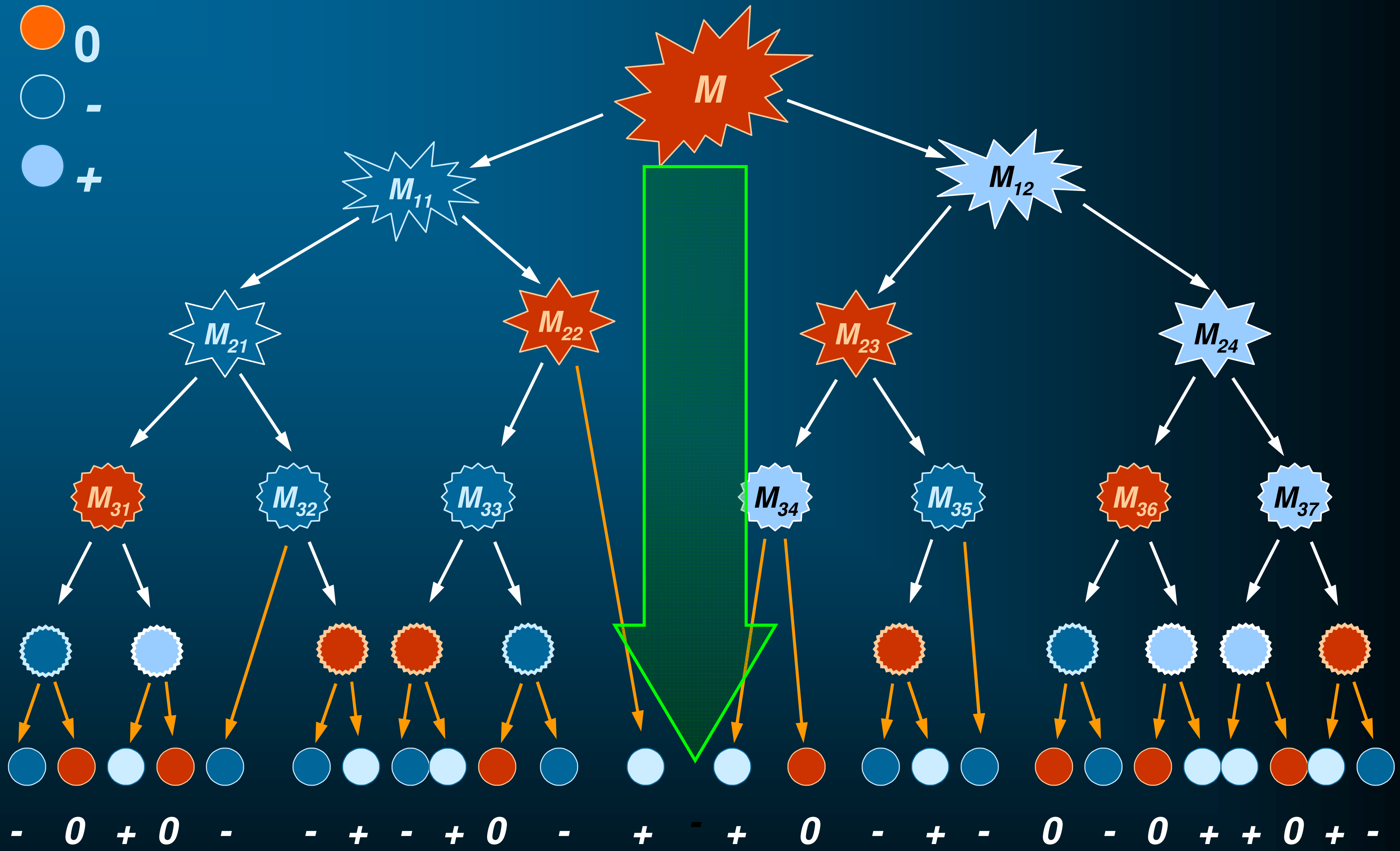
Results ...

$$C_2 = \frac{N_2^{BEC}(p_1,p_2)}{N_2^{ref}(p_1,p_2)}$$

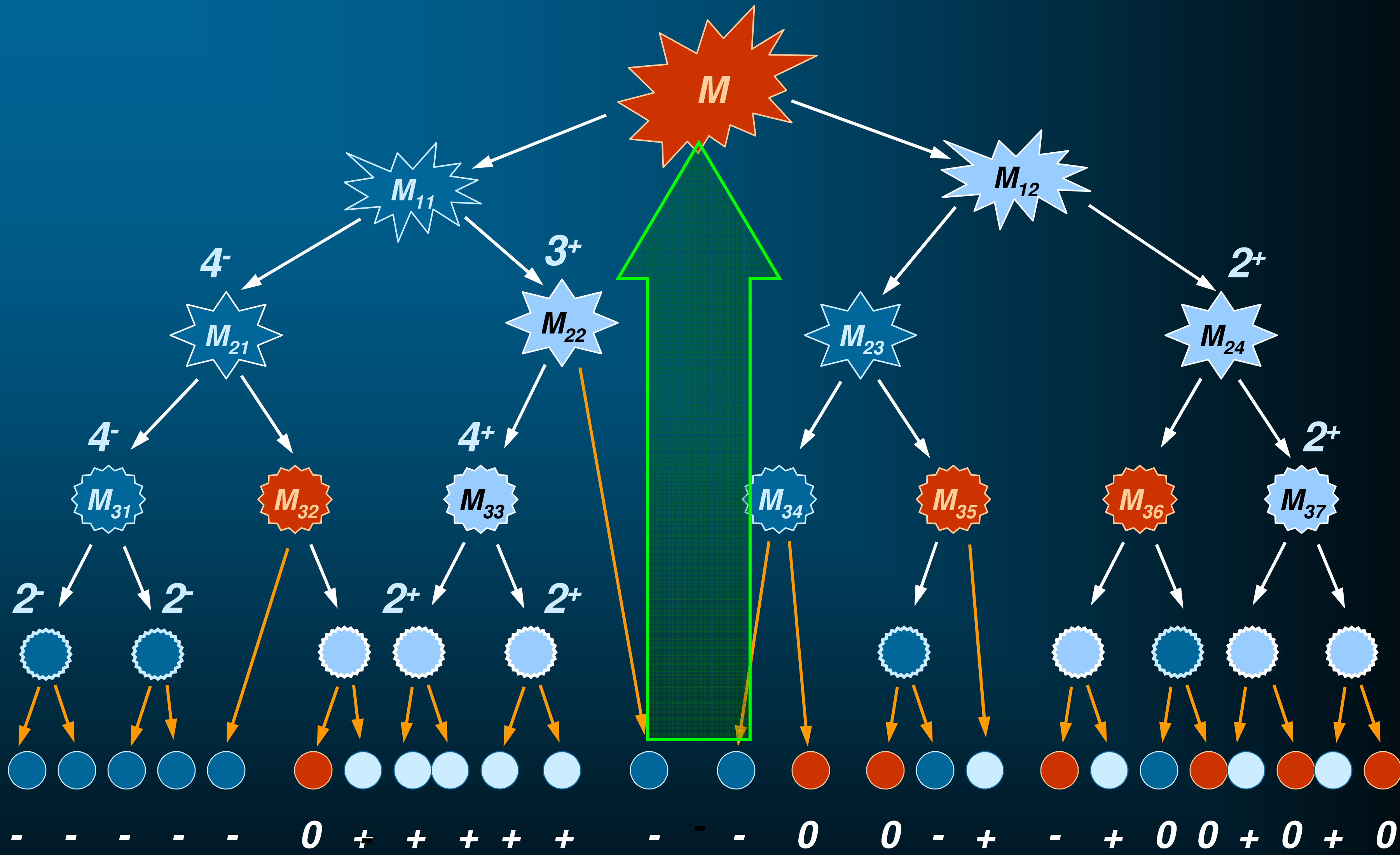
$$F_q(\delta y) = \frac{M^{q-1}}{\langle N\rangle^q}\left\langle \sum_{m=1}^{M} n_m(n_m-1)\cdots(n_m-q+1)\right\rangle$$



Cascade – charge flow …



Cascade (leading to BEC) – charge flow … reconstruction



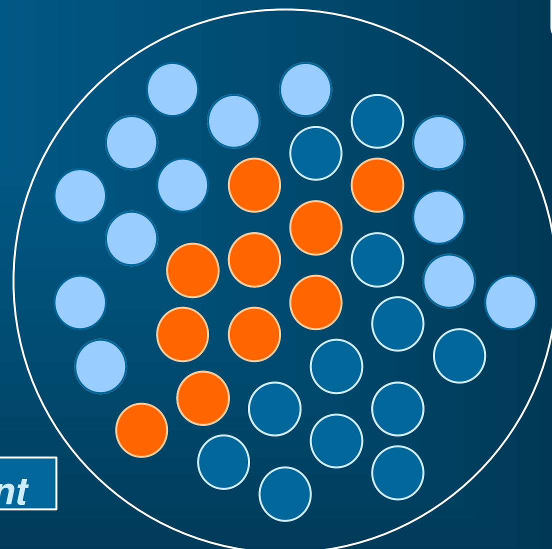
Summary (cascade)

- we conserve energy-momenta, charges, ...
- we preserve the shape of P(n)....

Momenta shifting

Weighting of events

single event



weight

$$P = P_0 e^{-f(p)g(x)}$$

change sign assignment

- *dynamical information is modified....*

  for example: anticorrelations between (+) and (-) are introduced ....

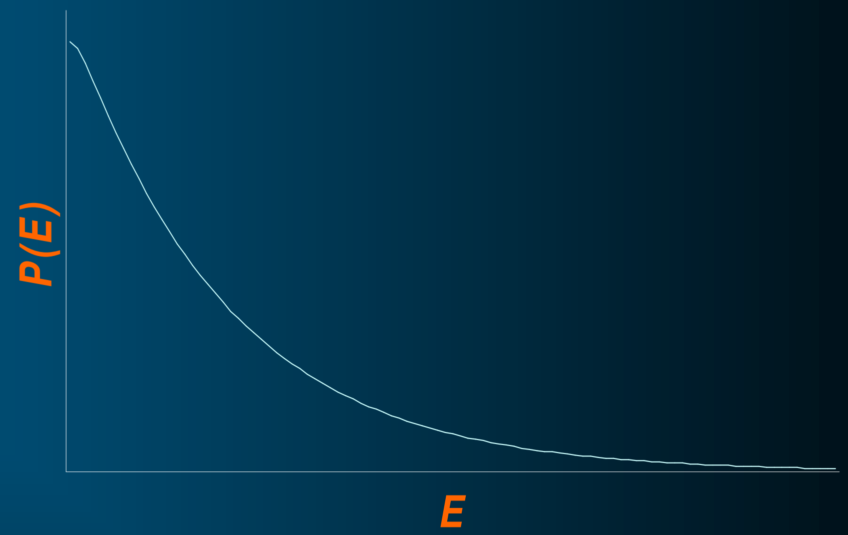
* OU, G.Wilk, Z.Włodarczyk, *Phys.Lett.* **B522** (2001) 273 and *Acta Phys.Pol.* **B33** (2002) 2681

Bose-Einstein … *II*

- Choose particles one-by one according to

$$f(E) = e^{-\frac{E}{kT}}$$

as long as energy allows



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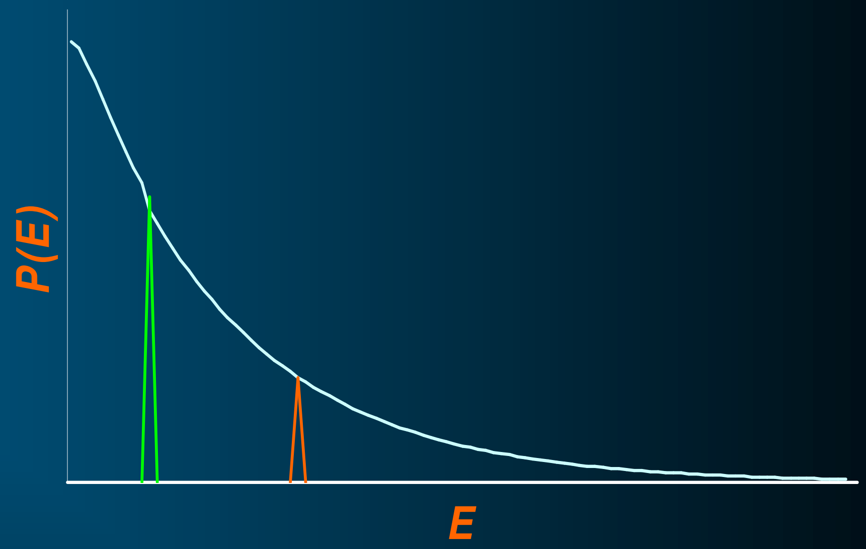

- Treat it as a *SEED* for a cell of particles (P_{cell} = *POISSON*)

Bose-Einstein … *II*

- Choose particles one-by one according to

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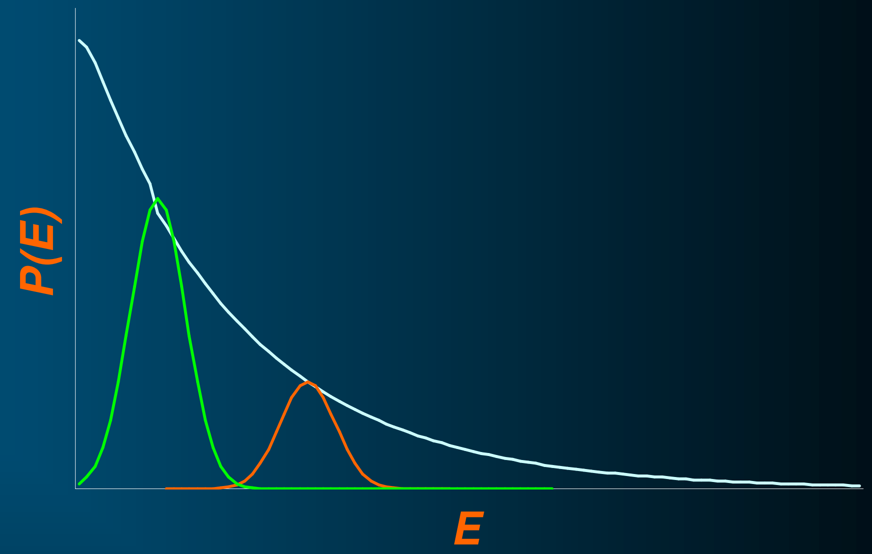


- Treat it as a *SEED* for a cell of particles (P_{cell} = *POISSON*)
- add (with probability *P* until first failure) to it particles of the same charge *Q* and energy *E*

$$g(E) \propto e^{-\frac{(E-E_0)^2}{2\sigma_E^2}}$$

Why smear energy* …

$V \to \infty$	$e^{-ip^{\mu}x_{\mu}}$	$\left[\hat{c}(p_{\mu}),\hat{c}^{+}(p'_{\nu})\right]=\delta^{4}(p_{\mu}-p'_{\nu})$	$C_2(Q)=1+\delta(QR)$
$V=V_0$	$e^{-\frac{p^2}{2\sigma_P^2}}$	$\left[\hat{c}(p_{\mu}),\hat{c}^{+}(p'_{\nu})\right]=\Delta^{4}(p_{\mu}-p'_{\nu})$	$C_2(Q)=1+f(QR)$

* G.A.Kozlov, OU, G.Wilk, *Phys.Rev.* **C68** (2003) 024901 and *Ukr. J. Phys.* **48** (2003) 1313

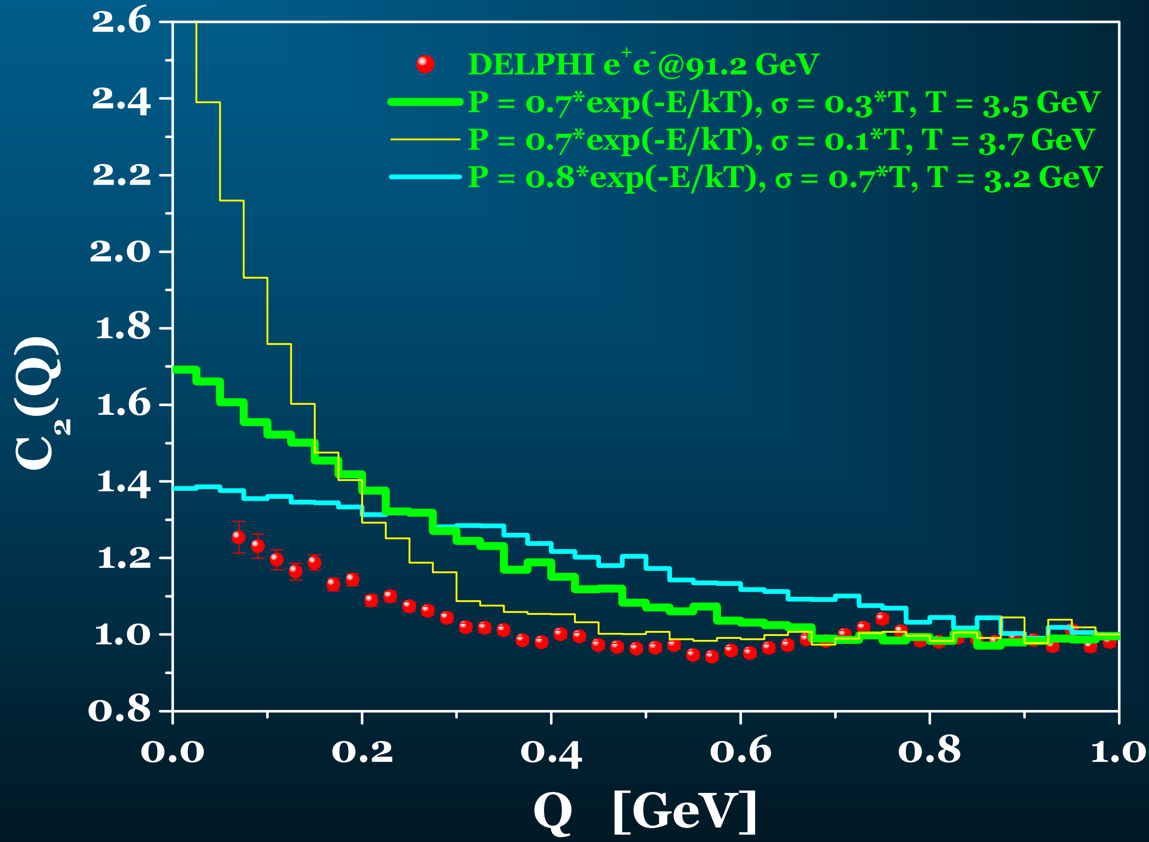
Results ...

Parameters (1st order ...)

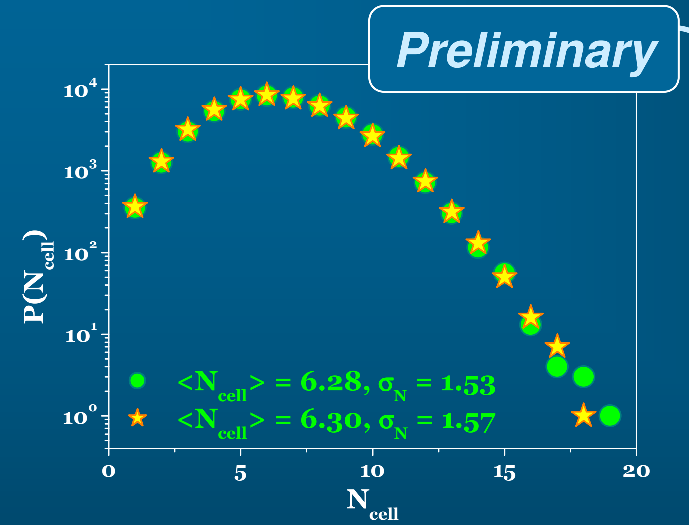
T	$\Leftrightarrow$	$\langle N_{ch} \rangle$
P_0	$\Leftrightarrow$	$\lambda \equiv C_2(Q=0)-1$
$g(E)$	$\Leftrightarrow$	***shape of*** $C_2(Q)$

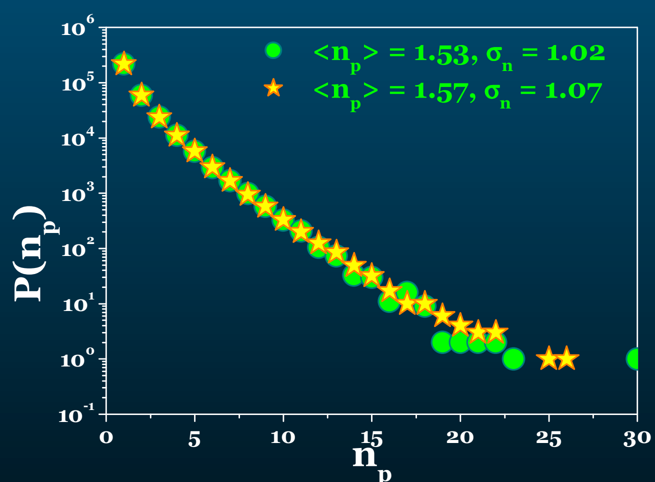
Results ... (for time being ...)

$$C_2(Q) = \frac{N_2^{BEC}(p_1, p_2)}{N_2^{Boltzmann}(p_1, p_2)}$$

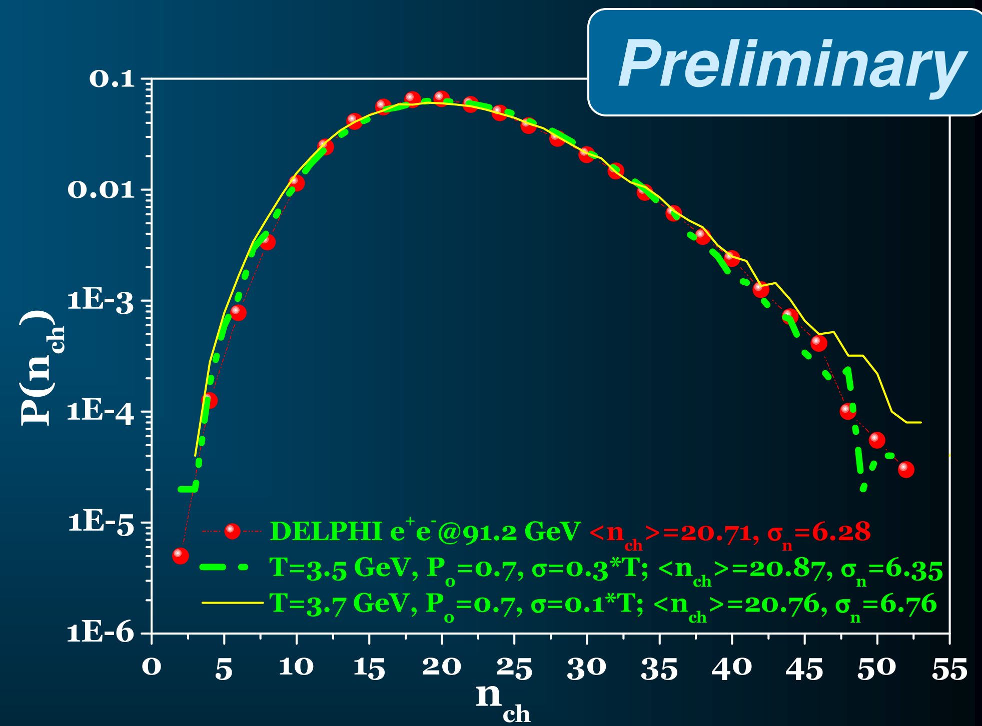


Results ...

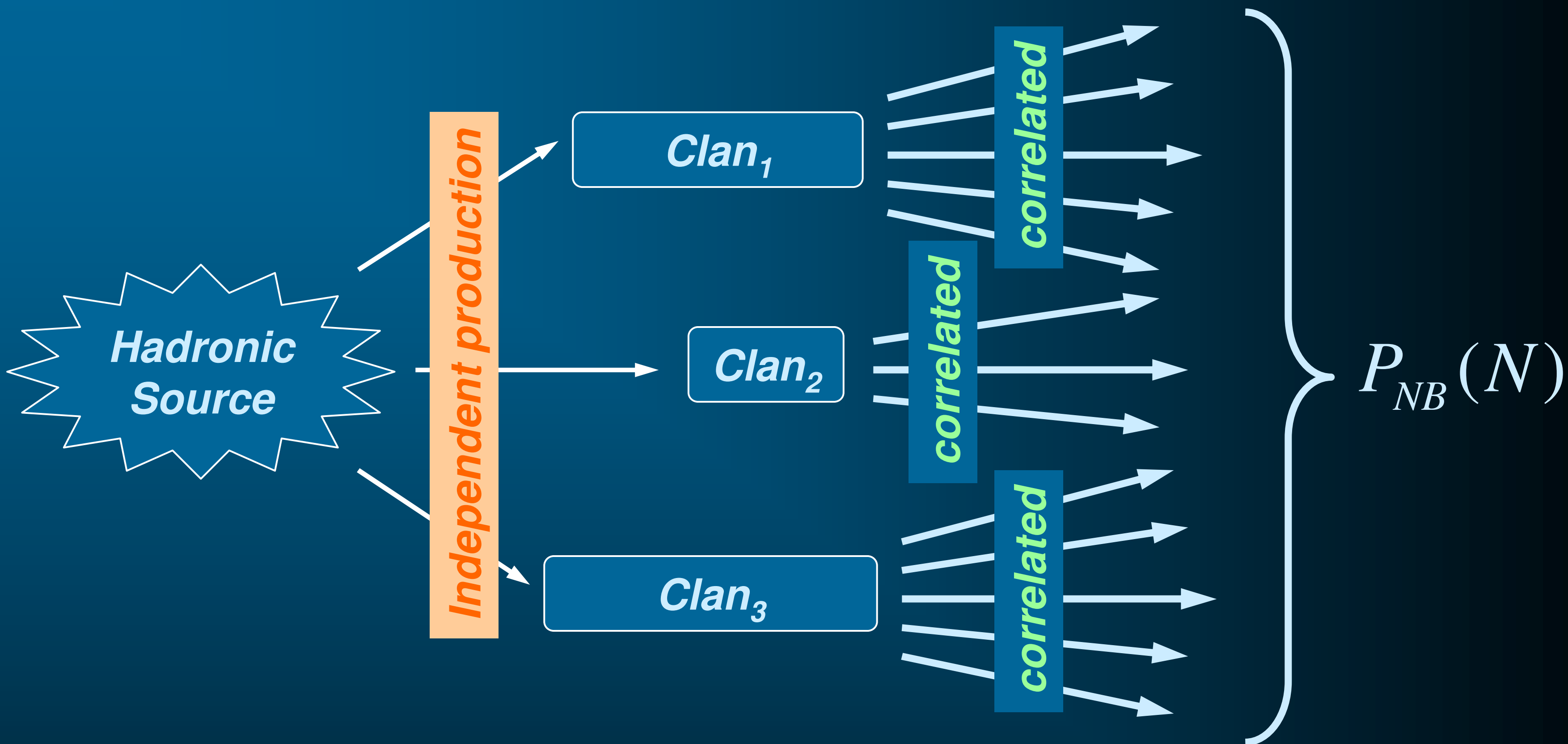




$$P(N_{cell}) \otimes P(n_{part})$$



Clan model*



$$P_{Possion}(N_{Clan}) \otimes P_{\log}(N_{part})$$

* L. Van Hove and A. Giovannini, *XVII Int. Symp. On Mult. Dyn.*, ed. by M.Markitan (World Scientific, Singapore 1987), p. 561

Clan model … (MD)

- $$P_{Poisson}(N_{cell}) = \frac{\nu^{N_{cell}}}{N_{cell}!} e^{-\nu} \otimes P_{Logarithm}(n_{part}) \propto \frac{b^{n_{part}}}{n_{part}}$$

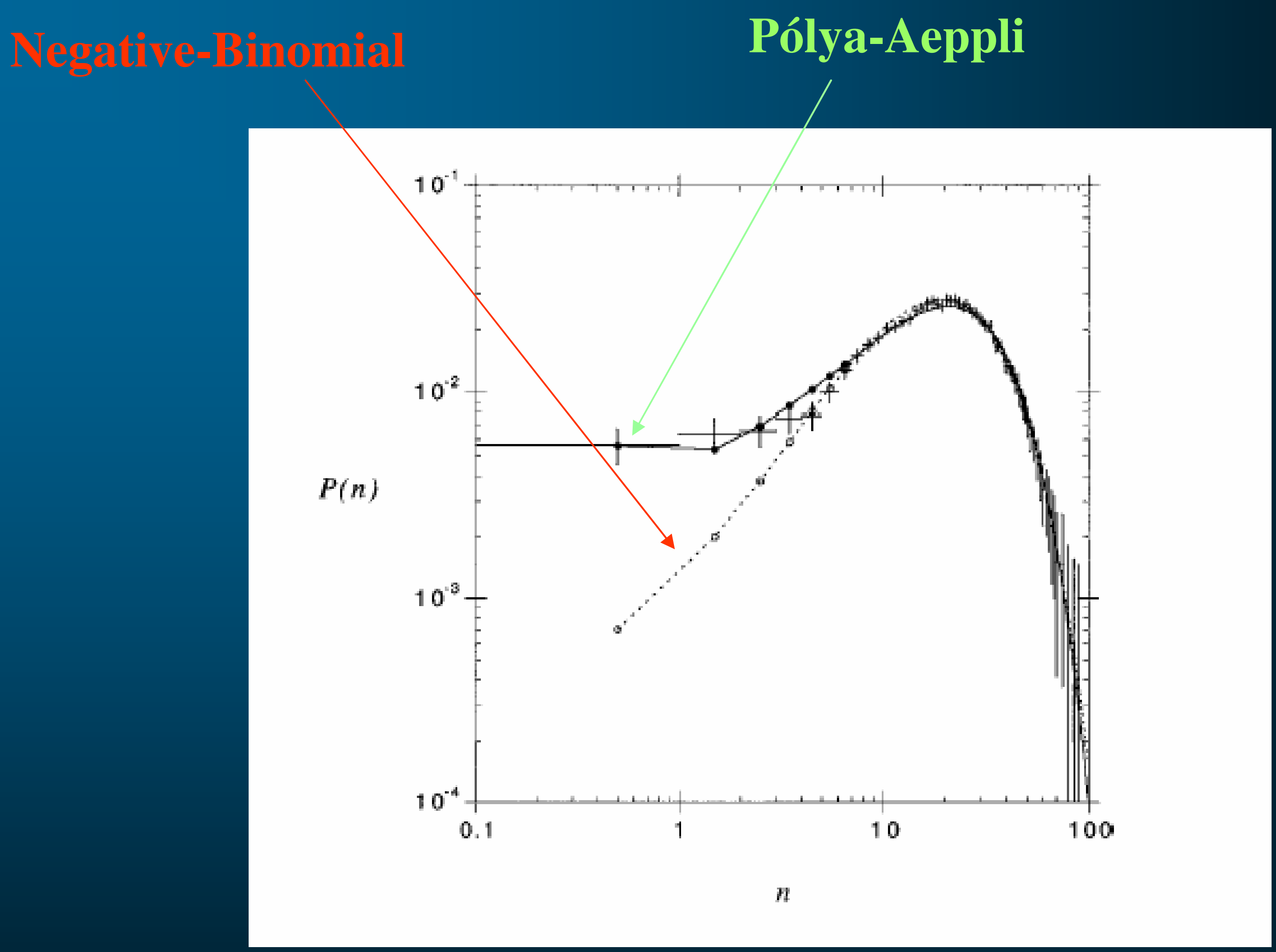
Negative Binominal multiplicity distribution (NBD)

Quantum statistics*

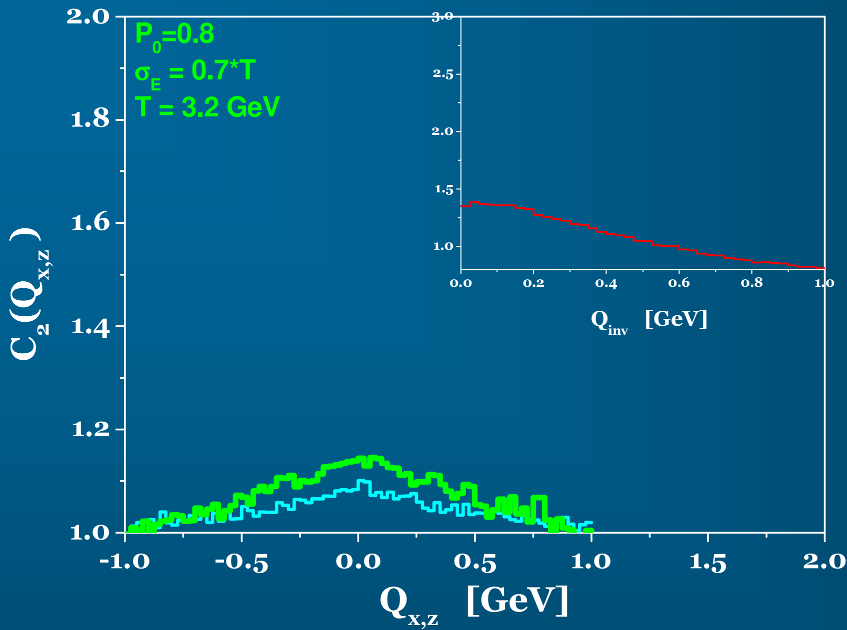
- $$P_{Poisson}(N_{cell}) = \frac{\nu^{N_{cell}}}{N_{cell}!} e^{-\nu} \otimes P_{Geometric}(n_{part}) \propto b^{n_{part}}$$

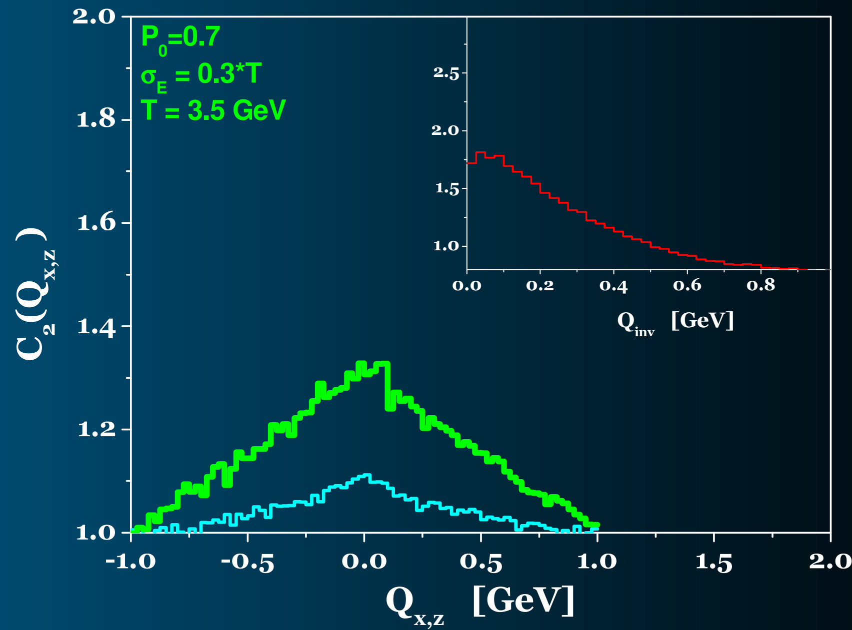
Pólya-Aeppli multiplicity distribution (PAD)

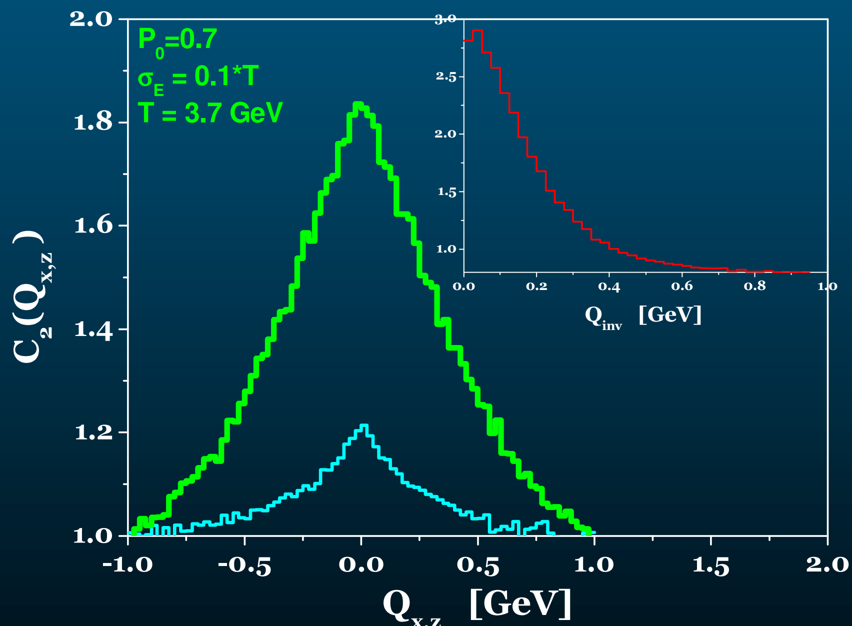
* J.Finkelstein, *Phys. Rev.* **D37** (1988) 2446 and Ding-wei Huang, *Phys. Rev.* **D58** (1998) 017501



* Ding-wei Huang, *Phys. Rev.* **D58** (1998) 017501







$$\cos\Theta :\Leftarrow f(\cos\Theta) = e^{-\frac{1}{2\sigma_\Theta^2}(1-\cos\Theta)^2}$$

$$\langle p_T \rangle = 0.3, \sigma_\Theta = 0.1$$

Summary and conclusions

- ***Our aim: to obtain*** $C_2(Q) > 1$ ***directly from MC event generator TOGETHER with*** $P(n_{ch})$***, intermittency,*** $\frac{1}{N_{ch}}\frac{dN_{ch}}{dy}, \ldots$

- ***Notice that to get*** $C_2(Q) > 1$ ***and correct shape one HAS TO introduce smearing of the momentum in the cell (clan). This is similar (equivalent?) to:***

  - ✓ ***replacing*** $\delta(Q) \Leftrightarrow$ ***strongly peaked function as in QFT approach to BEC****

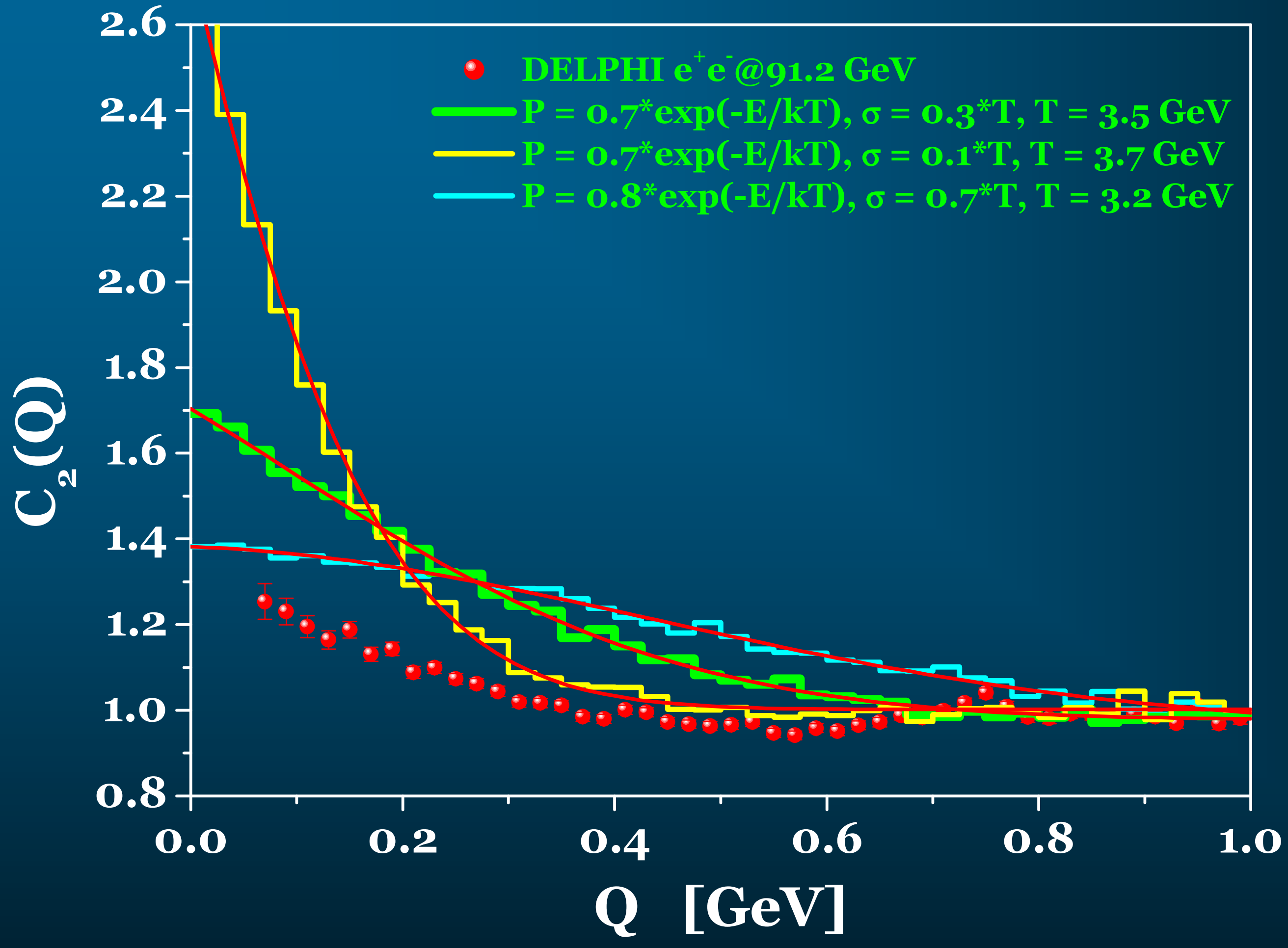
  - ✓ ***introducing source function being a Fourier transform of this function***

* G.A.Kozlov, OU, G.Wilk, *Phys.Rev.* **C68** (2003) 024901 and *Ukr. J. Phys.* **48** (2003) 1313

Summary and conclusions

- ***Our proposition seems to work on simple example. It remains to be checked whether it will work:***
  - ✓ ***with resonances included;***
  - ✓ ***with more complicated scenarios for $f(E)$ function (for example: including flows,***
    many sources,
    final-state interactions ,
    ....).

Back-up Slides



$$\sigma_E = 2.24 \Rightarrow \sigma_{fit} = 0.95$$

$$\sigma_E = 1.05 \Rightarrow \sigma_{fit} = 0.74$$

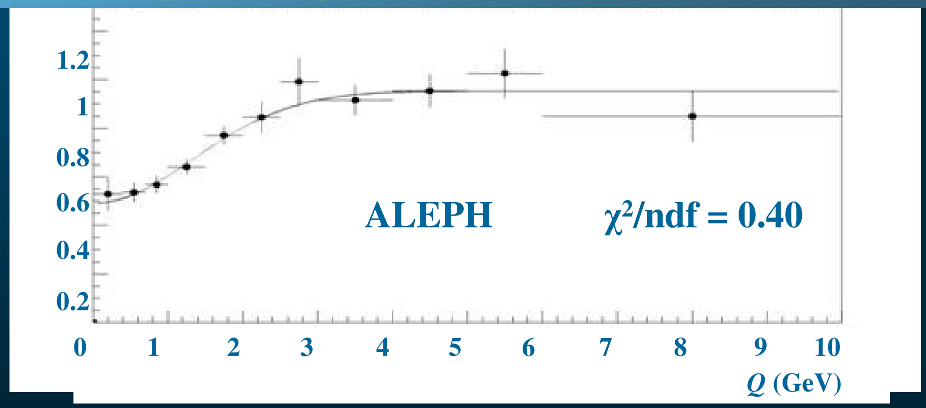
$$\sigma_E = 0.37 \Rightarrow \sigma_{fit} = 0.46$$

FDC Data

$$C_2(Q=|p_1-p_2|)\equiv\frac{N_2^{FD}(p_1,p_2)}{N_2^{ref}(p_1,p_2)}$$

ALEPH (sub. to Phys.Lett. B): FDC in pp and $\bar{p}\bar{p}$ pairs

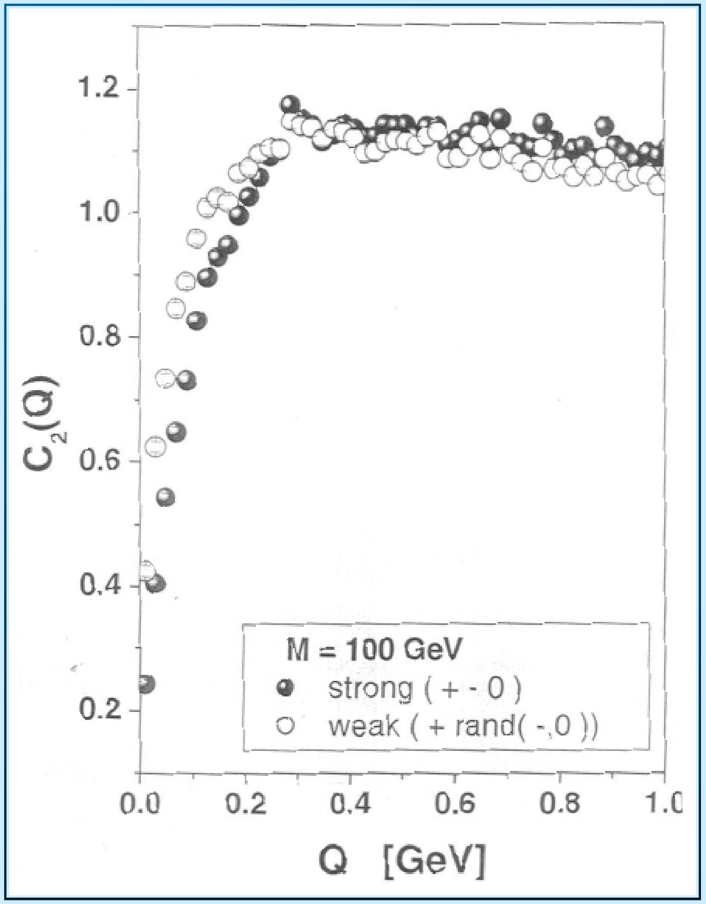
- 3526 pp,$\bar{p}\bar{p}$ pairs in the region 0 < Q < 10 GeV
- 74% of pairs from the primary vertex for Q < 5 GeV
- 65% of pairs for Q < 10 GeV



M. Kucharczyk, hep-ex/0405057

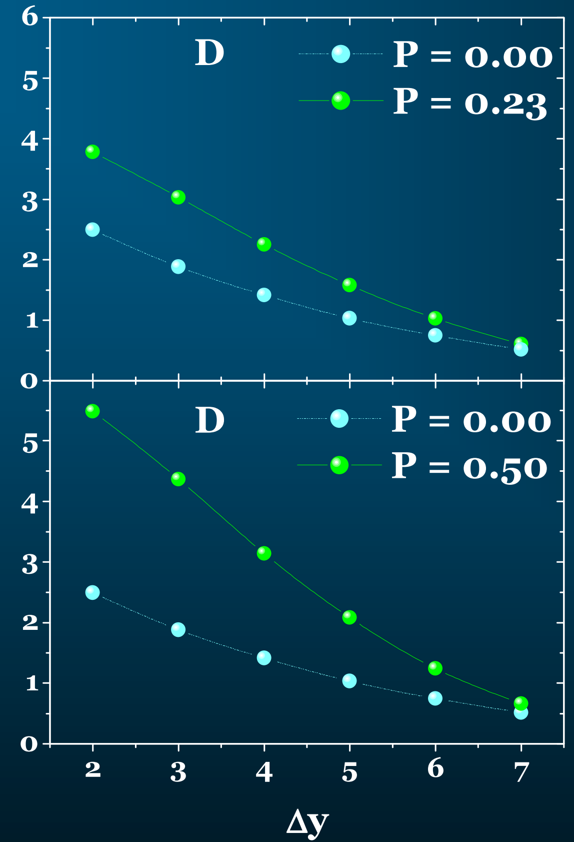
Results … (FD)

$$C_2 = \frac{N_2^{FEC}(p_1, p_2)}{N_2^{ref}(p_1, p_2)}$$



Charge fluctuations ...

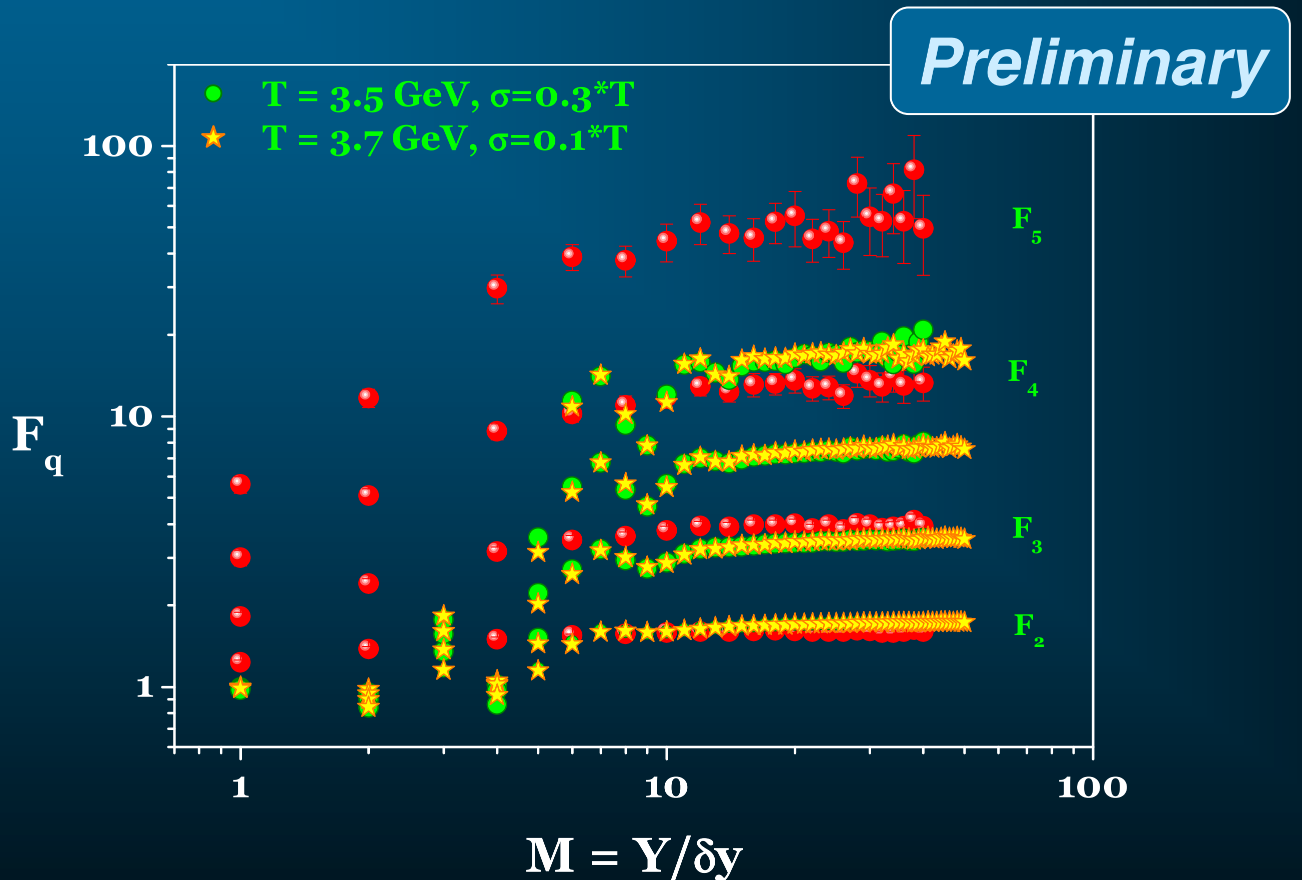
$$D \equiv \frac{\langle \delta Q^2 \rangle}{\langle N_{ch} \rangle}\ ^{*}$$



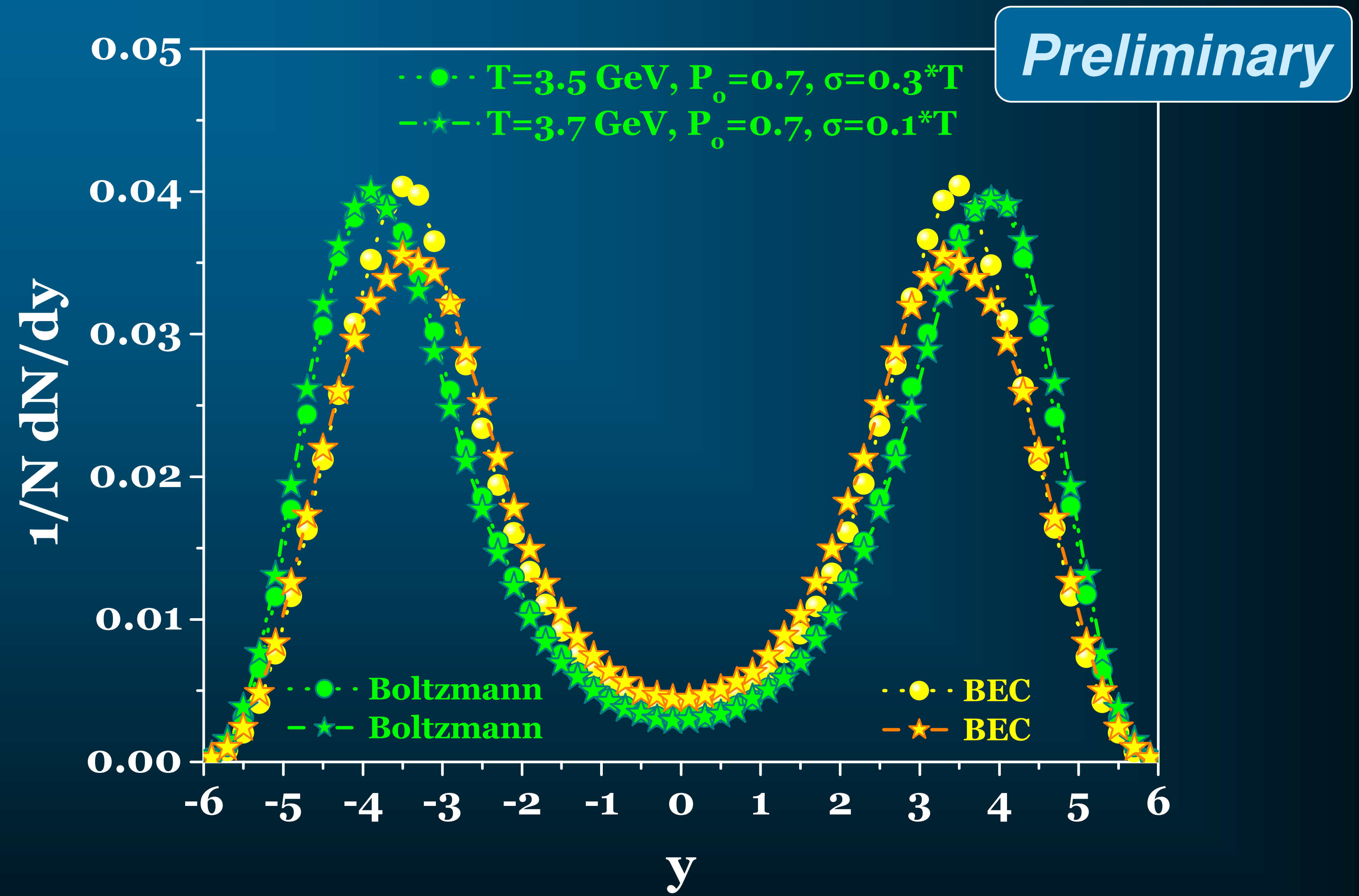
* M.Döring and V.Koch, *Acta Phys. Polon.* **B33** (2002) 1495, (nucl-th/0204009)

Results

$$F_q(\delta y) = \frac{M^{q-1}}{\langle N \rangle^q} \left\langle \sum_{m=1}^{M} n_m (n_m - 1) \cdots (n_m - q + 1) \right\rangle$$



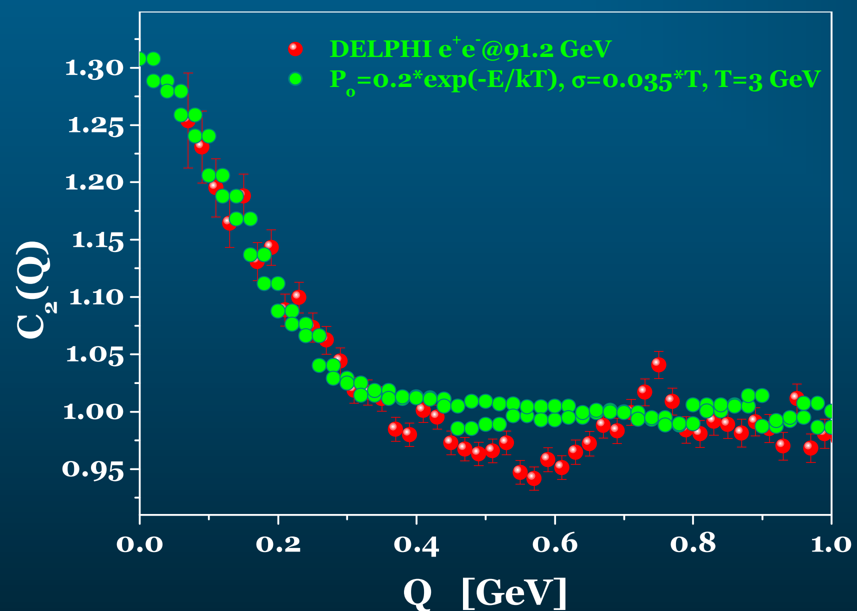
Results ...

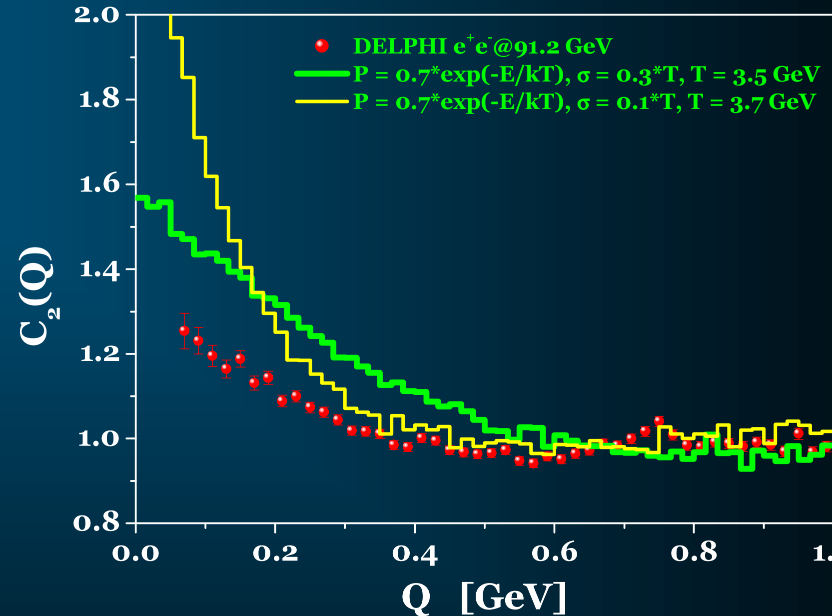


Preliminary
T=3.5 GeV, P_o=0.7, σ=0.3*T
T=3.7 GeV, P_o=0.7, σ=0.1*T
Boltzmann
Boltzmann
BEC
BEC
1/N dN/dy
y
0.05
0.04
0.03
0.02
0.01
0.00
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6

Old results ...

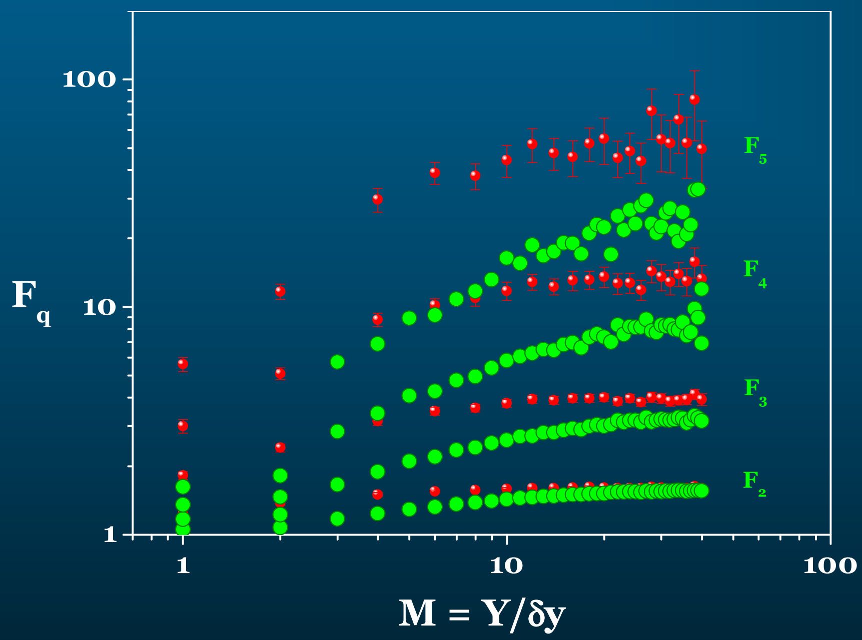
$$P = 0.2e^{-\frac{E}{kT}}, \sigma_E = 0.035T, T = 3GeV$$

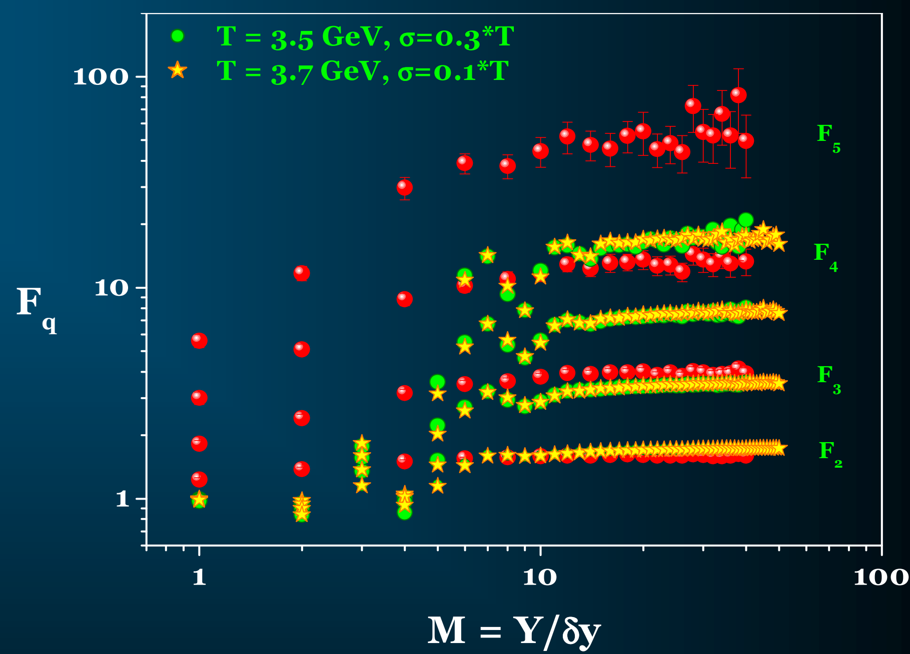




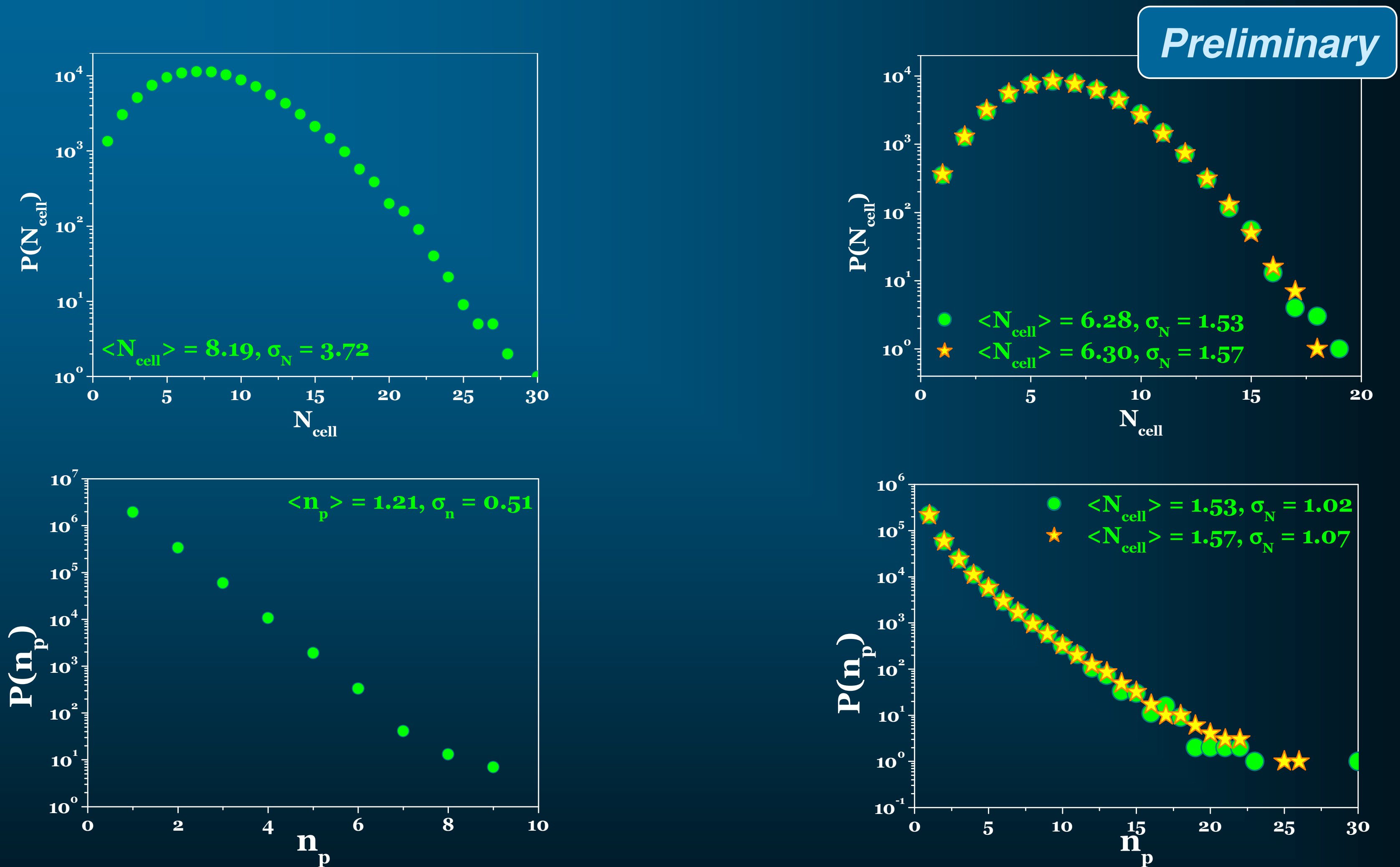
Old results …

$$P = 0.2e^{-\frac{E}{kT}}, \sigma_E = 0.035T, T = 3GeV$$



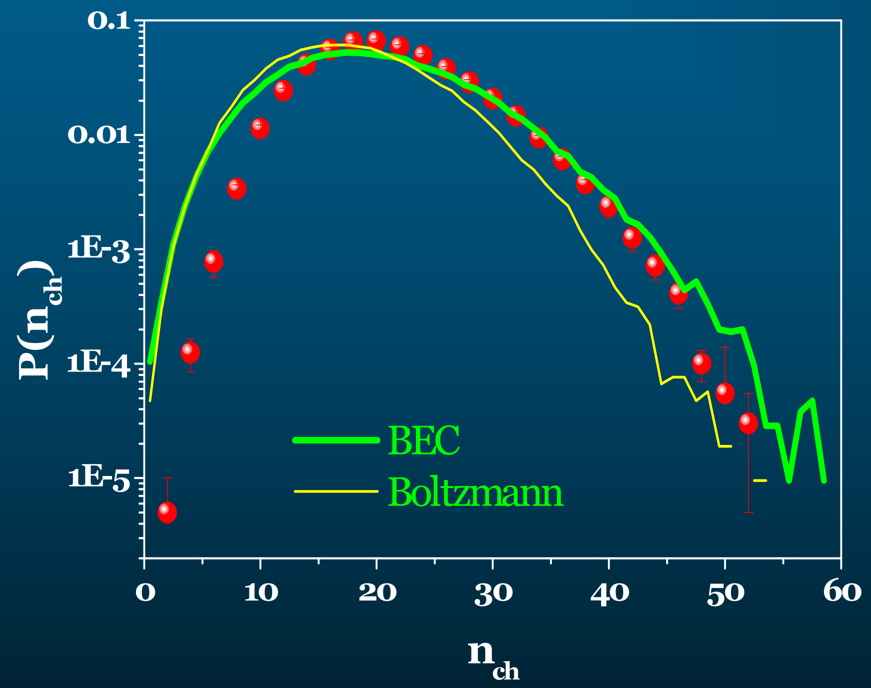


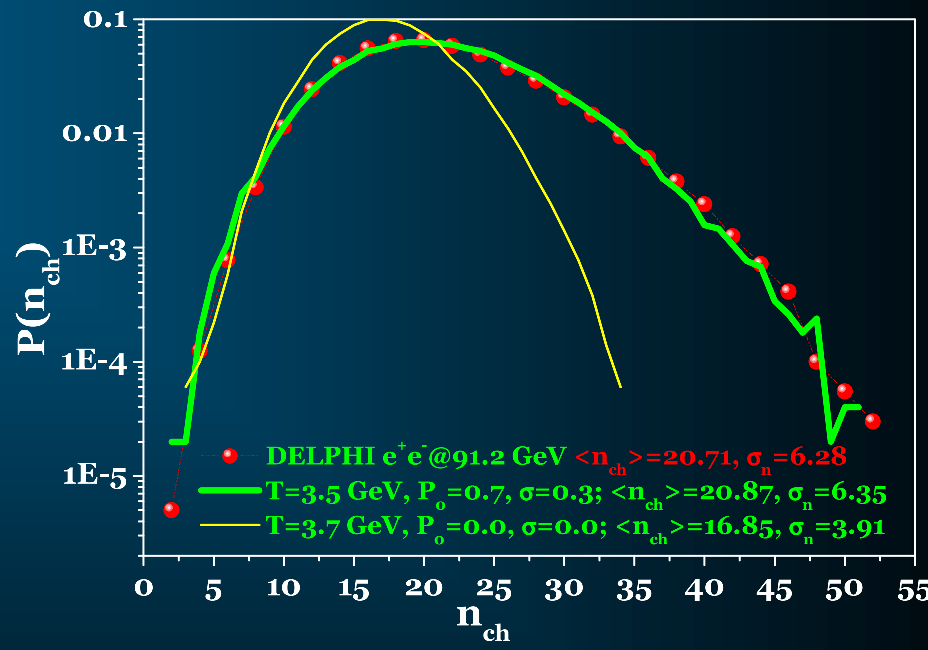
Results ...


Preliminary
P(N_cell)
N_cell
<N_cell> = 8.19, σ_N = 3.72
<N_cell> = 6.28, σ_N = 1.53
<N_cell> = 6.30, σ_N = 1.57
P(n_p)
n_p
<n_p> = 1.21, σ_n = 0.51
<N_cell> = 1.53, σ_N = 1.02
<N_cell> = 1.57, σ_N = 1.07

Old results ...

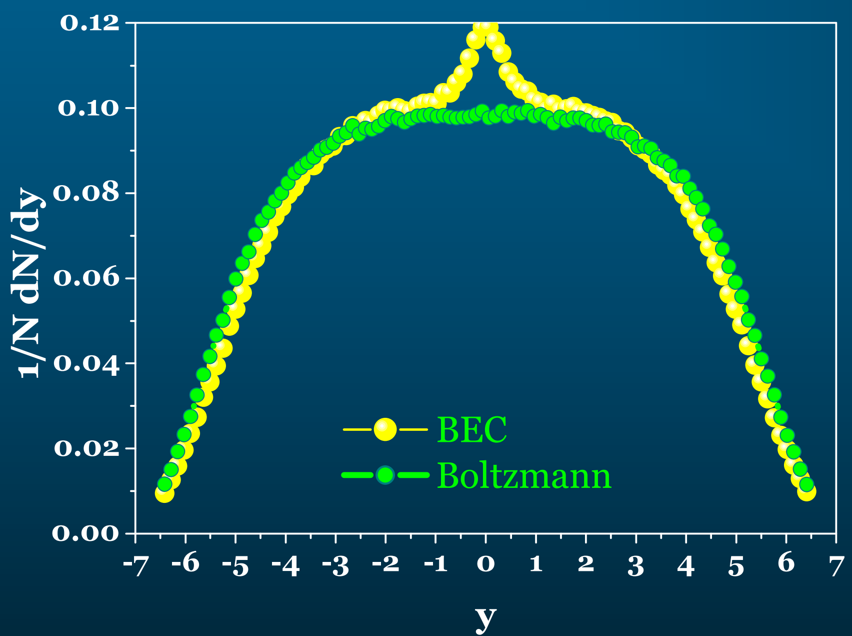
$$P = 0.2e^{-\frac{E}{kT}}, \sigma_E = 0.035T, T = 3GeV$$

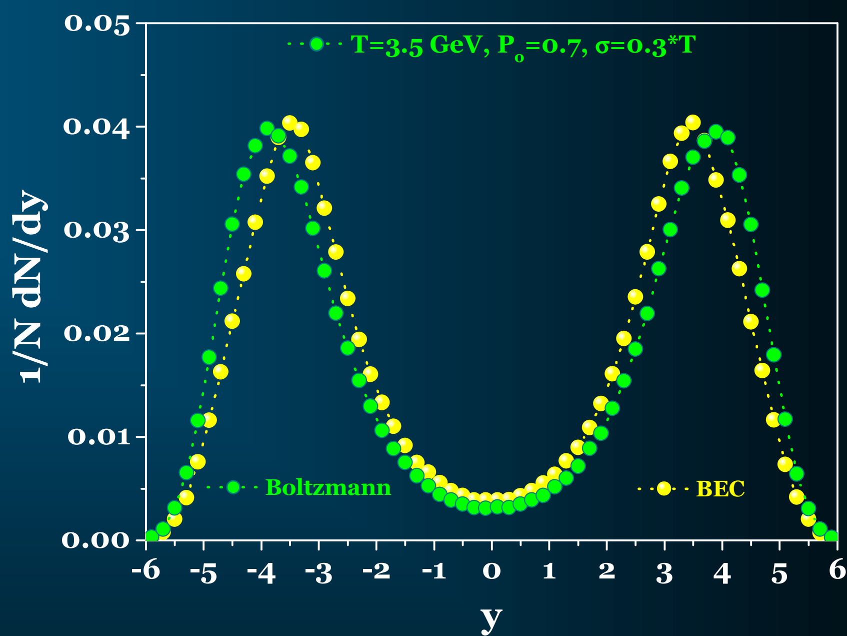




Old results …

$$P = 0.2e^{-\frac{E}{kT}}, \sigma_E = 0.035T, T = 3GeV$$



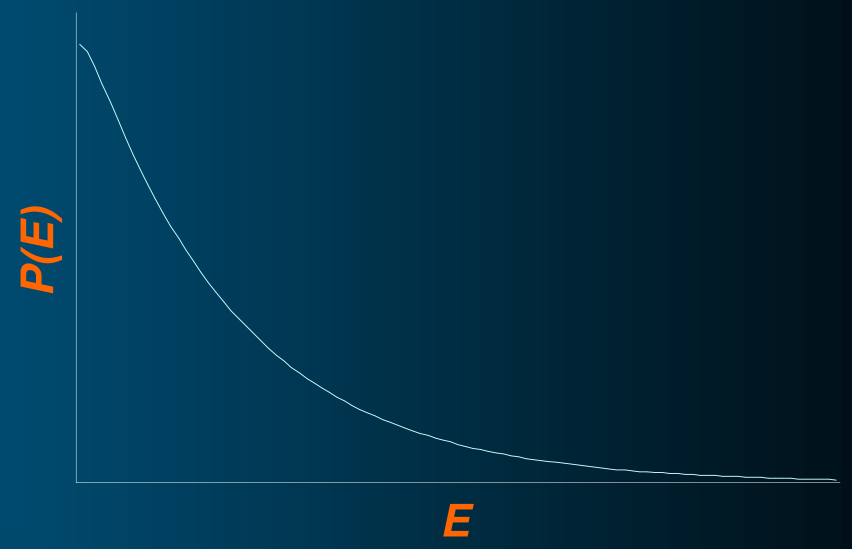


Boltzmann …

- Choose particles one-by-one according to

$$f(E) = e^{-\frac{E}{kT}}$$

as long as energy allows



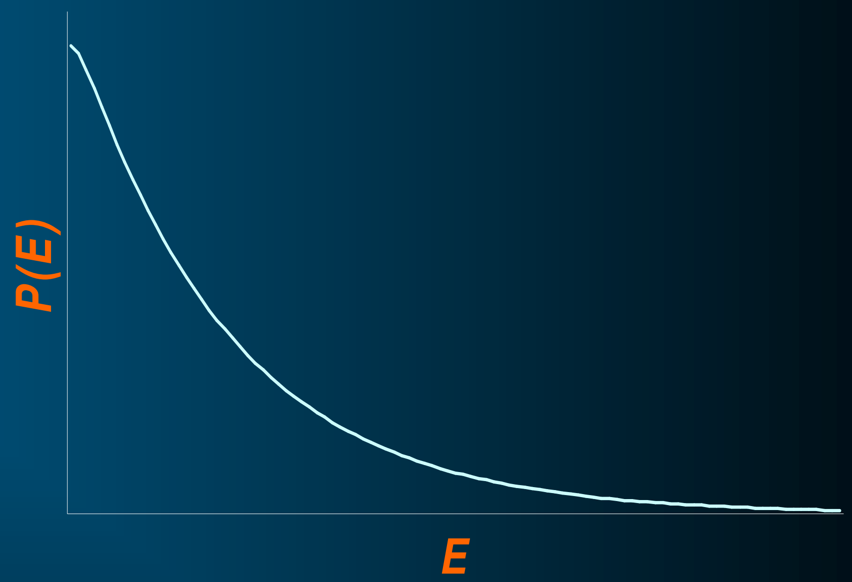
- Choose randomly charge *Q* {+,-,0}
- Correct for *E*, *p* and *Q* conservation

Bose-Einstein … *I*

- Choose particles one-by one according to

$$f(E) = e^{-\frac{E}{kT}}$$

as long as energy allows



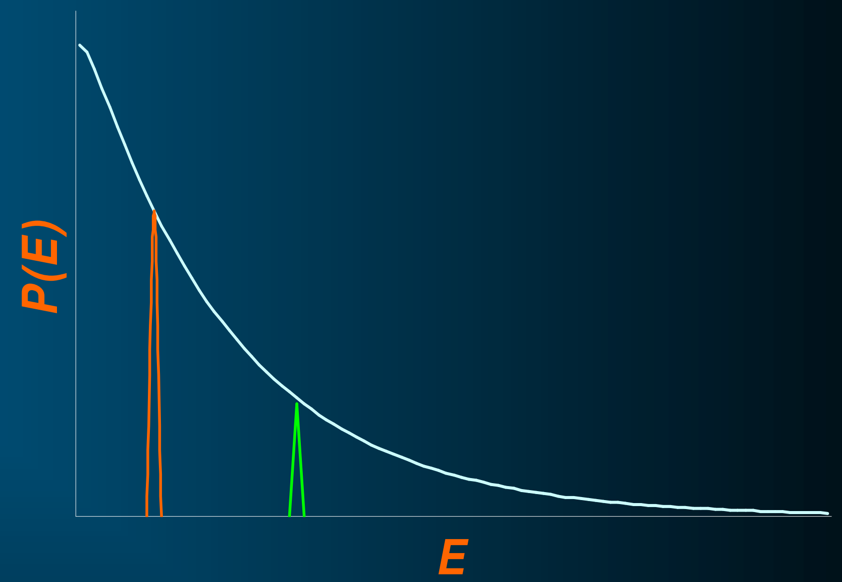
- Treat it as a *SEED* for a cell of particles (P_{cell} = *POISSON*)

Bose-Einstein ... *I*

- Choose particles one-by one according to

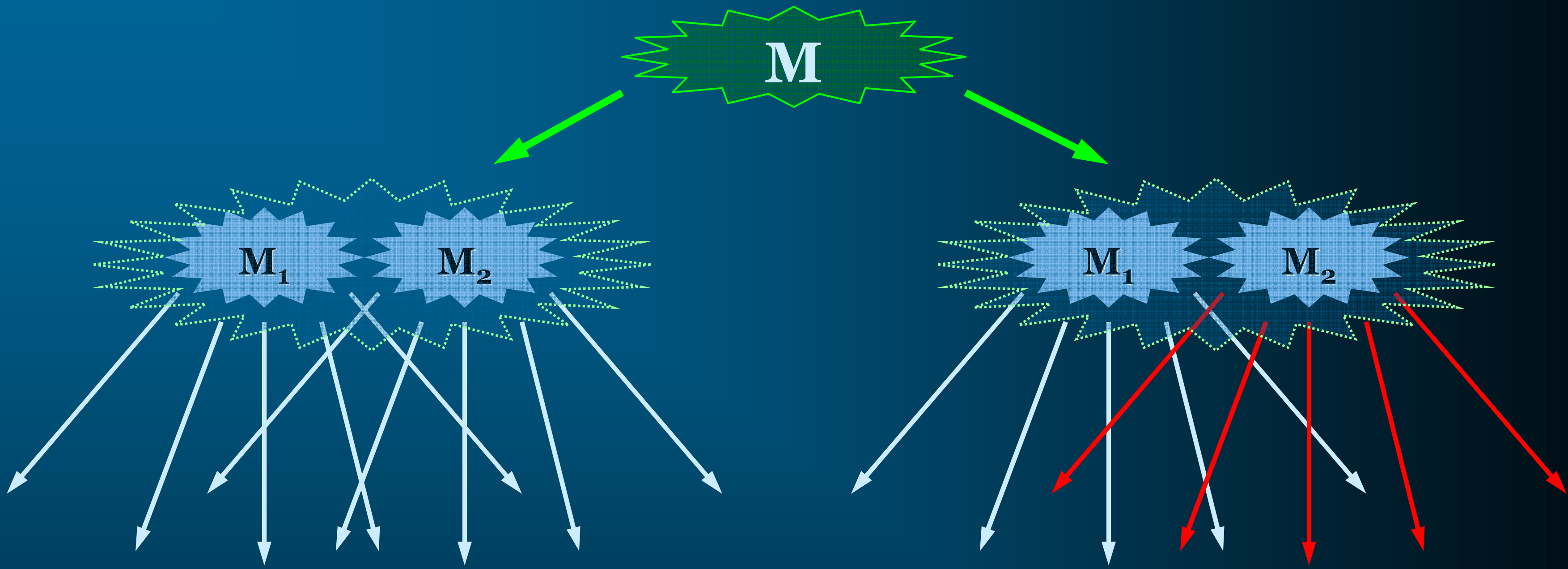
$$f(E) = e^{-\frac{E}{kT}}$$

as long as energy allows



- Treat it as a *SEED* for a cell of particles (P_{cell} = *POISSON*)
- add to it particles of the same charge *Q* and energy *E* with probability $P = P_0 f(E)$ until first failure

Results ...



Measurement of inter-W BEC signal

Measurement of inter-W BEC signal

Two observables:

$$\Delta\rho(Q) = \rho^{\mathrm{WW}} - 2\rho^{\mathrm{W}} - 2\rho^{\mathrm{WW}}_{\mathrm{mix}}$$

$$D(Q) = \frac{\rho^{\mathrm{WW}}}{2\rho^{\mathrm{W}} + 2\rho^{\mathrm{WW}}_{\mathrm{mix}}}$$

Mixed method: $\rho^{\mathrm{WW}}_{\mathrm{mix}} \approx \rho^{\mathrm{W}^+}\rho^{\mathrm{W}^-}$

Genuine inter-W correlation function:

$$\delta_{\mathrm{I}}(Q) = \frac{\Delta\rho(Q)}{2\rho^{\mathrm{WW}}_{\mathrm{mix}}(Q)}$$

Indication for inter-W BEC:

$$\Delta\rho(Q) \neq 0$$

$$D(Q) \neq 1$$